Performance Measures of a UWB Multiple-Access System: DS/CDMA versus TH/PPM

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Abstract

Time-hopping combined with pulse position modulation has been the original proposal for ultra-wideband systems. Spread spectrum techniques for multiple access and interference suppression are being considered for ultra-wideband radio systems. A digital impulse radio receiver using direct-sequence spreading is proposed in the paper and the signal processing scheme is investigated. Performance measures, such as error rate, multi-user capability, and system data rate for the proposed receiver structure, are derived mathematically and compared with the impulse receiver deploying time-hopping. The direct-sequence format is shown to support more active users; i.e, provide multi-access communications at a lower bit-error rate. Upper bounds for the number of permissible active users and the system data rate are derived analytically.

1 Introduction

In the near future, there will appear demand for high-speed wireless links for short range communication. A typical scenario could be in a building or an office environment. One family of techniques that allows for the creation of such high-rate, short-range systems is called ultra wideband. Ultra wideband is a flavor of spread spectrum communications characterized by transmitting narrow radio impulses (on the order of nanoseconds) [1], [2]. In this case, a signal is transmitted with a bandwidth much larger than the data modulation bandwidth and thus with a reduced power spectral density. This approach has the potential to produce a signal that is more covert, has higher immunity to interference effects, and has improved time-of-arrival resolution. It should also be noted that the gigahertz bandwidth implies that the multipath is resolvable to the path delays on the order of nanoseconds. This significantly reduces fading effects even in indoor environments [3].

To enhance its multiple access (MA) capability, UWB impulse radio systems can be deployed combining them with spread spectrum (SS) techniques. Time-hopping (TH) and direct-sequence (DS) are popular and simple schemes considered for UWB systems. A lot of research done in this area has been devoted to the TH-SS UWB signals. The time-hopping scheme combined with spread spectrum (PPM) and its variations such as the M-ary PPM and M-ary PPM using Walsh codes are investigated and analyzed in [4]-[8]. Traditional direct sequence spread spectrum has been studied extensively, and various forms of modulation such as binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK), can be employed with this spreading scheme. In [9], the rake receiver performances for the DS and TH schemes are compared in the presence of narrowband interference and dense multipath via simulations. TH is a part of the original proposal for UWB communications [4]-[5], but as shown in [10], DS is an established technique for combating multi-user interference (MAI) on AWGN channels and will outperform PPM. So it is vital to understand and study the DS-UWB impulse radio system.

In this paper, a digital impulse response radio receiver for a direct-sequence scheme is proposed and its multiple access capability is discussed and compared with that of the TH/PPM impulse radio UWB system. The signal processing in the receiver is analyzed and its performance in a MA environment with respect to the number of active users and system data rate are investigated. The system bit error probability (BER) expression is derived in detail and the MA capability of the system as a function of the number of users is studied. The performance measures derived serve in analytically interpreting the ability of the UWB system to support MA communications. Upon comparing the impulse radio technology using the two schemes, it is seen that DS/CDMA will outperform TH/PPM in terms of the SNR performance and can support a larger number of users while providing a better of quality of service.

The organization of the paper is as follows. In Section II, the DS-UWB signal transmission scheme and receiver structure are presented. This is followed by the SNR calculations for the receiver. In Section III, the key performance measures such as the BER and bounds for the number of active users supported by the system and the maximum achievable system data rate are derived. Finally, conclusions are presented in Section IV.

2 DS/CDMA UWB Multiple-Access Scheme

2.1 Transmission Signal Format

The basic transmitted CDMA waveform of user k is given by

$$c_{tr}^{(k)}(t) = \sum_{n=0}^{N_c-1} p_n^{(k)} w_{tr}(t - nT_c),$$

where w_{tr} represents the transmitted monocycle and $\{p_n^{(k)}\}$ denotes the pseudo-random noise (PN) sequence. Denote the period of the PN sequence of every user as N_c. Let T_f be the symbol period and T_c be the chip period such that $T_f = N_c T_c$. Hence, a typical DS format of the kth impulse radio transmitter output signal is given by

$$s_{tr}^{(k)}(t) = \sqrt{P_k} \sum_j d_j^{(k)} c_{tr}^{(k)}(t - jT_f), \tag{1}$$

where $\{d_j^{(k)}\}$ represents the data symbols and P_k is the transmitted power corresponding to the kth user. It is important to point out that even an ideal channel and antenna system modify the shape of the transmitted monocycle $w_{tr}(t)$ to $w_{rec}(t)$ at the output of the receiver antenna. The w_{rec} considered here is the second derivative of a Gaussian function and is given by $w_{rec}\left(\frac{t}{t_n}\right) = \left[1 - 4\pi\left(\frac{t}{t_n}\right)^2\right] \exp\left[-2\pi\left(\frac{t}{t_n}\right)^2\right]$ where t_n denotes a time normalization factor. The autocorrelation function for the monocycle is given by

$R\left(\frac{x}{t_n}\right) = \left[1 - 4\pi\left(\frac{x}{t_n}\right)^2 + \left(\frac{4\pi^2}{3}\right)\left(\frac{x}{t_n}\right)^4\right] \exp\left[-\pi\left(\frac{x}{t_n}\right)^2\right]$. For simplicity, we assume that the receiver has knowledge of this modified pulse shape. Let $s_{rec}^{(k)}$ be the received version of the waveform transmitted by the kth user. Finally, if we consider N_u transmitters, the aggregate transmitted waveform at the output of the receiver antenna looks like

$$r(t) = \sum_{k=1}^{N_u} \sqrt{P_k}\beta_k s_{rec}^{(k)}(t - \tau_k) + n(t), \tag{2}$$

where β_k captures the channel attenuation of the kth user signal upon propagation, τ_k represents the time asynchronism between the signal received from the kth transmitter, and $n(t)$ is additive white Gaussian noise with constant power spectral density σ_n^2. Without loss of generality, we consider user 1 to be the desired user for the purpose of analysis.

2.2 Receiver Signal Processing

The Digital Impulse Radio Multiple-Access receiver (DIRMA) is a form of correlator-matched filter and is based on the theory of hypothesis testing for fully coherent data detection. For the purpose of analysis, it is assumed that the receiver has perfect clock and sequence synchronization for the signal transmitted by the first transmitter, as a result of which, we know the inputs for the frame clock module. The template generated at the receiver again makes use of the assumption that the receiver has knowledge of the modified pulse shape of the monocycle.

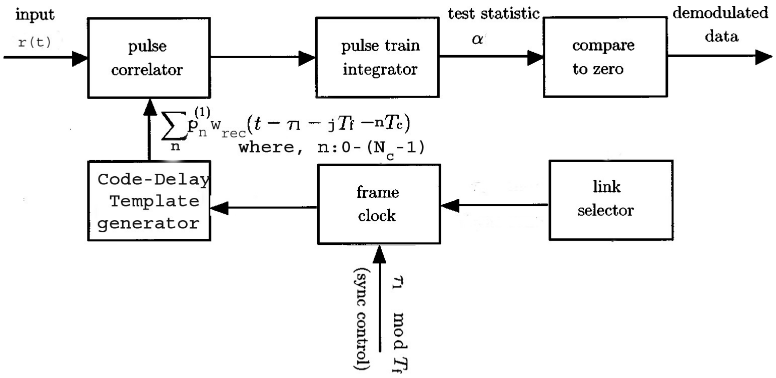

Figure 1: DIRMA Receiver Structure.

Without loss of generality, let us consider the situation where we are detecting the lth bit. Hence, the template generator for our scheme is different from the ones described in [4] and [5] and is given by $c_{rec}^{(1)}(t - lT_f - \tau_1)$, where $c_{rec}^{(1)}(t) = \sum_{n=0}^{N_c-1} p_n^{(1)} w_{rec}(t - nT_c)$. The DIRMA receiver must decide if $d_l^{(1)}$ is 1 or -1. This corresponds to deciding between two hypotheses $\mathcal{H}_{-1}$ and $\mathcal{H}_1$.

$$\mathcal{H}_{d_l} \ : \ r(t) = \beta_1\sqrt{P_1}\ d_l^{(1)} c_{rec}^{(1)}(t - lT_f - \tau_1) + n_{tot}(t) \tag{3}$$

in which d_l is either -1 or 1. The other terms are grouped as shown below.

$$n_{tot}(t) = \underbrace{\sum_{k=2}^{N_u} \sqrt{P_k}\beta_k s_{rec}^{(k)}(t-\tau_k)}_{\text{Multiple-Access Noise}} + \underbrace{n(t)}_{\text{Additive White Gaussian Noise}}$$

$$''\text{decide } \; d_l^{(1)} = -1'' \;\Leftrightarrow\; \underbrace{\int_{\tau_1+lT_f}^{\tau_1+(l+1)T_f} r(t)\, c_{rec}^{(1)}(t-lT_f-\tau_1)\,dt}_{\text{pulse correlator output}} > 0. \tag{4}$$

Strictly speaking, the decision rule presented is not optimum when other users are present. Now, if we make the assumption that the number of users is large, it is reasonable to model the MAI as a Gaussian random process as done in [4], [5], [9], and [10]. Thus $n_{tot}(t)$ is a white Gaussian random process (since it is the sum of two Gaussian random processes) and equation (4) is optimum. For this problem, the pulse correlator output (or the test statistic) α can be rewritten as $\alpha = m + \tilde{n}$, where m and $\tilde{n}$ are given by

$$\begin{aligned} m &= \int_{\tau_1+lT_f}^{\tau_1+(l+1)T_f} \left[\beta_1\sqrt{P_1} d_l^{(1)} c_{rec}^{(1)}(t-lT_f-\tau_1)\right] c_{rec}^{(1)}(t-lT_f-\tau_1)\,dt, \quad \text{and} \\ \tilde{n} &= \int_{\tau_1+lT_f}^{\tau_1+(l+1)T_f} n_{tot}(t)\, c_{rec}^{(1)}(t-lT_f-\tau_1)\,dt. \end{aligned} \tag{5}$$

Equation (5) can be decomposed into the term due to the MAI and AWGN as

$$\begin{aligned} n_{MAI} &= \int_{\tau_1+lT_f}^{\tau_1+(l+1)T_f} \sum_{k=2}^{N_u} \sqrt{P_k}\beta_k s_{rec}^{(k)}(t-\tau_k)\, c_{rec}^{(1)}(t-lT_f-\tau_1)\,dt, \quad \text{and} \\ n_{noise} &= \int_{\tau_1+lT_f}^{\tau_1+(l+1)T_f} n(t)\, c_{rec}^{(1)}(t-lT_f-\tau_1)\,dt. \end{aligned} \tag{6}$$

Now, the quantity m, is mathematically derived to be

$$m = N_c A_1 d_l^{(1)}\xi, \quad \text{where} \;\; \xi = \int_{-\infty}^{\infty} [w_{rec}(t)]^2\,dt \;\; \text{and} \;\; A_1 = \sqrt{P_1}\beta_1. \tag{7}$$

3 Comparison of Performance Measures

3.1 SNR Computations

The DIRMA receiver output SNR is defined as

$$SNR_{out}(N_u) = \frac{m^2}{E\{|\tilde{n}|^2\}},$$

where the numerator in this expression is given by (7). Without much effort, it can be shown that the denominator, $E\{|\tilde{n}|^2\} = \sigma_{noise}^2 + N_c\sigma_{self}^2\sum_{k=2}^{N_u} A_k^2$, where $\sigma_{self}^2 = T_f^{-1}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} w_{rec}(x-s)w_{rec}(x)dx\right]^2 ds$ and $A_k = \sqrt{P_k}\beta_k$. In (6), it is assumed that the mean and variance of $n_{noise}(t)$ are given by 0 and σ^2_{noise} respectively. Then the DIRMA receiver output SNR is mathematically given by

$$SNR_{out}(N_u) = \frac{(N_c A_1 \xi)^2}{\sigma^2_{noise} + N_c \sigma^2_{self} \sum_{k=2}^{N_u} A_k^2}. \tag{8}$$

When only the desired transmitter is active, $N_u = 1$, and the expression for single-user output SNR is $SNR_{out}(N_u = 1) = \frac{(N_c A_1 \xi)^2}{\sigma^2_{noise}}$.

3.2 Error Probability and BER Comparisons

The output SNR in (8) can also be expressed as

$$SNR_{out}(N_u) = \left\{ SNR_{out}^{-1}(1) + \left[\frac{N_c \xi^2}{\sigma^2_{self}}\right]^{-1} \sum_{k=2}^{N_u} \left(\frac{A_k}{A_1}\right)^2 \right\}^{-1}. \tag{9}$$

Now the expression for the probability of error turns out to be

$$P_e(N_u) = Q\left[\left\{ SNR_{out}^{-1}(1) + \left[\frac{N_c \xi^2}{\sigma^2_{self}}\right]^{-1} \sum_{k=2}^{N_u} \left(\frac{A_k}{A_1}\right)^2 \right\}^{\frac{-1}{2}} \right]. \tag{10}$$

When only the desired transmitter is active, $N_u = 1$, and the expression for single-user probability of error is $P_e(N_u = 1) = Q\left[\left\{\frac{(N_c A_1 \xi)^2}{\sigma^2_{noise}}\right\}^{\frac{1}{2}}\right]$. In the case of TH-UWB, the system is an oversampled modulation system with N_s monocycles transmitted per symbol, the modulation data changes only every N_s hops. In this analysis we assume $N_s = N_c$. From [4], [5], and [8], the correlator's template waveform regenerated at the DIRMA receiver is given by $v(t) = w_{rec}(t) - w_{rec}(t - \delta)$, where δ is the PPM time shift. The SNR expression is identical to (8) except that ξ is replaced by the correlation term, $m_p = \int_{-\infty}^{\infty} w_{rec}(t-\delta) v(t)\, dt$. Under the assumptions on the structure of w_{rec} (for example, the Gaussian monocycle), the autocorrelation function is non-negative. Thus, based on the choice of template generators described, $|m_p| < \xi$. Hence, upon comparing our results with those presented in [4] and [5], it is seen that

$$SNR_{out}^{(TH/PPM)}(N_u) < SNR_{out}^{(DS/CDMA)}(N_u) \;\Rightarrow\; P_e^{(TH/PPM)}(N_u) > P_e^{(DS/CDMA)}(N_u). \tag{11}$$

Another means of comparison which immediately follows is the usage of E_b/N_o, which is defined as

$$E_b/N_o(N_u) = SNR_{out}(N_u)[dB] + 10 \log_{10}(b+1), \tag{12}$$

when $b+1$ bits are transmitted per symbol. The theoretical BER is given by $BER = \frac{1}{2}\, erfc\left(\sqrt{\frac{SNR_{out}(N_u)}{2}}\right)$.

3.3 Transmission Capacity and Excess Single-Link SNR Comparisons

The excess single-link SNR, ΔP is given by $\Delta P = 10 \log_{10}\{SNR_{out}(1)/SNR_{out}(N_u)\}$. This is the additional power required to accommodate every new link in the multiuser system. This is one of the basic performance measures of a multiple-access system that relates number N_u of users and the $SNR_{out}(N_u)$. Using this definition, we can rewrite the sum in (9) as $\sum_{k=2}^{N_u}\left(\frac{A_k}{A_1}\right)^2 = M^{-1}SNR_{out}^{-1}(N_u)\{1-10^{(-\Delta P/10)}\}$ where, $M^{-1} = \frac{N_c\xi^2}{\sigma_{self}^2}$. Now, $M^{-1} = \frac{\xi^2}{T_f\sigma_{self}^2R_{mod}}$ where, we define $R_{mod} = (N_cT_f)^{-1}$ as the system data rate. Under perfect power control assumptions, i.e., $A_k = A_1$ for all k, and with $\tilde{M}^{-1} = \frac{\xi^2}{T_f\sigma_{self}^2}$, it reduces to $R_{mod}(\Delta P) = \tilde{M}^{-1}SNR_{out}^{-1}(N_u)\{1-10^{(-\Delta P/10)}\}\{N_u-1\}^{-1}$, which is the system data rate, providing a simple relation between the various parameters of interest. Following this analysis is the equation for the number of users as a function of the excess single-link SNR, given by $N_u(\Delta P) = \lfloor M^{-1}SNR_{out}^{-1}(N_u)\{1-10^{(-\Delta P/10)}\}\rfloor + 1$. Note that both the expressions for N_u and R_{mod} are monotonically increasing functions of ΔP. To fit a bound to these parameters, we let ΔP be as large as possible, i.e., $\Delta P \to \infty$ and the relations become

$$\begin{aligned} R_{mod}(\Delta P) &\leq \lim_{\Delta P\to\infty} R_{mod}(\Delta P) = \tilde{M}^{-1}SNR_{out}^{-1}(N_u)\{N_u-1\}^{-1} \equiv R_{max}, & (13)\\ N_u(\Delta P) &\leq \lim_{\Delta P\to\infty} N_u(\Delta P) = \lfloor M^{-1}SNR_{out}^{-1}(N_u)\rfloor + 1 \equiv N_{max}. & (14)\end{aligned}$$

Hence, for a specified level of performance as embodied in $SNR_{out}(N_u)$, there are upper bounds on the system data rates (for a given number of users) and the number of users (for a given system data rate) that cannot be exceeded by the DIRMA receivers. The performance measures, R_{max} and N_{max} are higher for the DS format and serve as a basis for comparison between the two formats for MA-UWB communication systems.

4 Conclusions

Ultra-wideband technology has been recently proposed as a viable solution for high-speed indoor short range wireless communication systems because of its robustness to severe multipath and multi-user conditions, low cost, and low power implementation. With this in view, an appropriate reception scheme for direct-sequence UWB impulse radio receiver was presented in the paper. The system multiple-access performance was analyzed in terms of the error probability, the data transmission rate, and number of active users supported under specified conditions. Expressions for the output SNR of the ultra-wideband system were derived and the BER performance of the receiver, as a function of the number of users, was seen to be superior to the time-hopping format. In general, this result was true for all signal (monocycle) waveforms which have non-negative autocorrelation functions. To conclude, with this choice of monocycle pulses, the DS-UWB supports larger number of users communicating at lower error rates and higher data transmission rates.

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