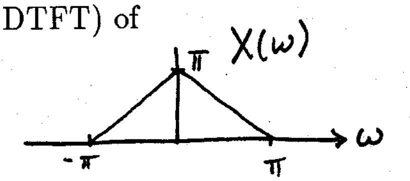
15 1. Find the function $y(n)$ that satisfies

$$y(n+2) + 6y(n+1) + 8y(n) = 5^n u(n), \qquad y(0) = 1, y(1) = 1.$$

Use the method of z transforms. There is a table on the last page of the exam.

15 2. Find the inverse discrete-time Fourier transform (inverse DTFT) of



Evaluate all integrals!!!

15 3. Let $h(n) = 2^n[u(n) - u(n-4)]$, and $x(n) = u(n) - u(n-8)$. Determine formulas for the discrete-time convolution, $(h * x)(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$ for $-\infty < n < \infty$.

4. For each of the following systems, state whether it is:
(linear/nonlinear), (TI/not TI), (causal/noncausal), (stable/unstable)

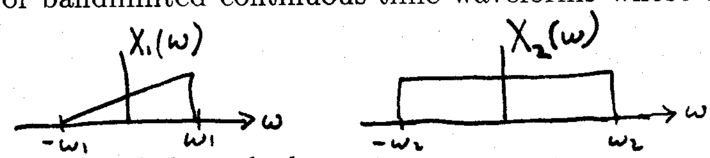
8 (a) $(Ax)(n) = \sum_{k=-\infty}^{\infty} \frac{\exp(\ln[1 + |x(n-k)|])}{1 + |k|}$

8 (b) $(Ax)(n) = \sum_{k=0}^{10} \frac{\sin(x(n-k))}{1+k}$

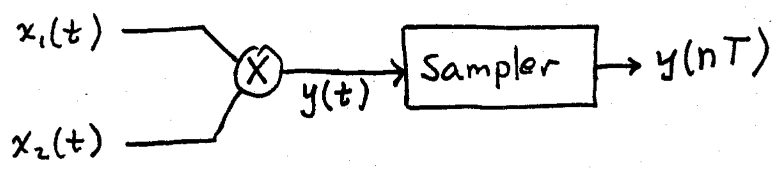
8 (c) $(Ax)(n) = n[x(n) - x(n+1)]$

15 5. Find the z-transform of $x(n) = n(n-1)a^n u(n)$, where $|a| < 1$.
Hint: You will need to derive a formula using the geometric series formula on the last page of the exam.

15 6. Let $x_1(t)$ and $x_2(t)$ be a pair of bandlimited continuous-time waveforms whose Fourier transforms are shown below.



Suppose these time waveforms are passed through the system:



In order to reconstruct $y(t) = x_1(t)x_2(t)$ from the samples $y(nT)$, what is the largest possible value of T according to the sampling theorem? JUSTIFY YOUR ANSWER!!! Recall that $T = 2\pi/\omega_s$, where ω_s is the sampling frequency.
Hint: Recall that

$$y(t) = x_1(t)x_2(t) \longleftrightarrow Y(\omega) = \frac{1}{2\pi}(X_1 * X_2)(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega - \nu)X_2(\nu)\,d\nu.$$