10 1. Find the function $y(n)$ that satisfies

$$y(n+2) + 4y(n+1) + 3y(n) = 2^n u(n), \qquad y(0) = 0, y(1) = 1.$$

Use the method of z transforms. There is a table on the last page of the exam.

15 2. For each of the following systems, determine whether it is *linear* or *nonlinear*:

(a) $(Tx)(n) = n$.

(b) $(Tx)(n) = \begin{cases} x(n), & \text{if } |x(n)| \leq 10, \\ 0, & \text{otherwise.} \end{cases}$

(c) $(Tx)(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{1+|k|}\right)^2 x(n-k)$.

(d) $(Tx)(n) = \ln\left(\prod_{k=0}^{3} e^{x(n+k)}\right)$.

(e) $(Tx)(n) = \begin{cases} x(n), & \text{for } n = 0, \ldots, 10, \\ 0, & \text{otherwise.} \end{cases}$

15 3. Let $X(\omega)$ be the discrete-time Fourier transform (DTFT) of $x(n)$. Suppose $X(\omega) = \omega$ for $-\pi \leq \omega \leq \pi$. Find $x(n)$ for all n. Evaluate all integrals.

15 4. For each of the following systems, determine whether it is *causal* or *noncausal*:

(a) $(Tx)(n) = \sum_{k=-1}^{1} 2^{-k} \cos(x(n-k-1))$.

(b) $(Tx)(n) = \sum_{k=-1}^{1} 3^{-k} |x(n \cdot k)|^2$.

(c) $(Tx)(n) = x(0) + \sum_{k=-\infty}^{n} 2^{k-n} x(k)$.

(d) $(Tx)(n) = \prod_{k=0}^{3} x(n-k)$.

(e) $(Tx)(n) = \sum_{k=-\infty}^{\infty} 7^{-|n-k|} x(k) u(k)$.

10 5. In Problem 3, $|x(n)| = 1/n$ for $n \neq 0$. Use this fact to find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$. JUSTIFY YOUR ANSWER!

15 6. Consider the LTI system $(Tx)(n) = \sum_{k=0}^{\infty} \frac{x(n-k)}{\ln([2+k]^2)}$.

Is this system stable? JUSTIFY YOUR ANSWER! *Hint:* $\ln\theta < \theta$ for $\theta > 0$.

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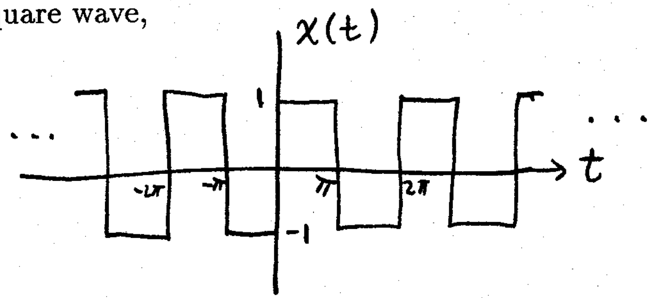
7. Let $h(t)$ be the impulse response of a continuous-time LTI system. Let $H(\omega)$ denote the corresponding transfer function.

(a) Let $x(t)$ have period T. If $y(t)$ is the system output,

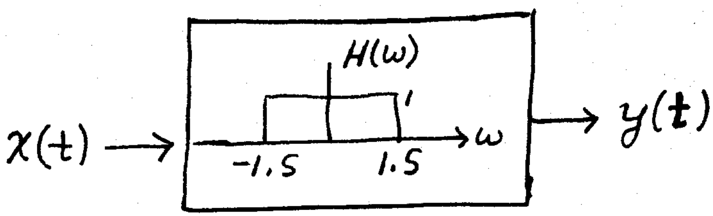
(i) show that $y(t)$ also has period T;

(ii) express the complex Fourier coefficients of y, say y_n, in terms of $H(\cdot)$ and x_n, where x_n is the nth Fourier coefficient of $x(t)$.

(b) When $x(t)$ is the square wave,



show that the output of the system



is $y(t) = \lambda \sin(t)$ for some constant λ. Do NOT try to find λ.