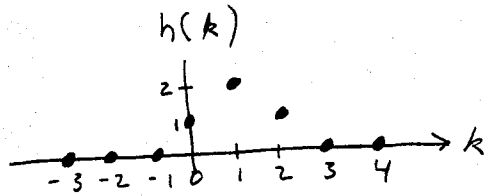
Closed book. Closed Notes. No Calculators.

20 1. Compute the convolution $(h * x)(n)$ for all n if $x(k) = \delta(k-1) + \delta(k-2)$ and $h(k)$ is shown below:



2. Consider the system $(Ax)(n) = \dfrac{x(n) - x(n-1)}{1 + |n+1|}$.

5 (a) Is A linear (Yes/No)?

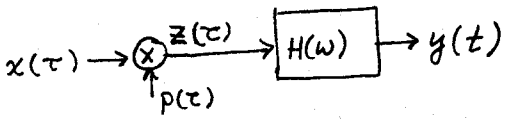
5 (b) Is A time invariant (Yes/No)?

5 (c) Is A causal (Yes/No)?

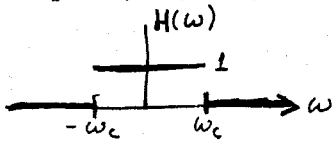
5 (d) Is A stable (Yes/No)?

20 3. Consider the system $(Ax)(n) = \dfrac{x(n+1) + x(n-1)}{2}$. Find the impulse response $h(n)$ and the transfer function $H(\Omega)$. **Evaluate all sums in closed form, and express $H(\Omega)$ as a real-valued function of Ω.**

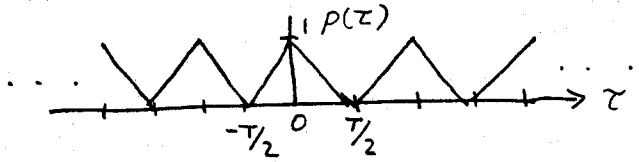
20 4. Consider the following system:



where x is a continuous-time, bandlimited signal with cut-off frequency ω_c, and p is periodic with period T. Suppose $H(\omega)$ is the ideal lowpass filter,



For what positive values of T will $y(t) = \lambda x(t)$ for some constant λ? Justify your answer, and find λ if p is the triangle wave



5. Let $x(k)$ and $y(k)$ be discrete-time signals.

10 (a) If $z(n) = \sum_{k=-\infty}^{\infty} x(n-k)y(k)$, show that the corresponding DTFTs satisfy $Z(\Omega) = X(\Omega)Y(\Omega)$.

10 (b) Find *all* values of the number E such that it is possible to find a discrete-time function $x(n)$ with both of the following two properties:

$$(i) \qquad \sum_{k=-\infty}^{\infty} x(n-k)x(k) = x(n), \quad \text{for all } n,$$

$$(ii) \qquad \sum_{n=-\infty}^{\infty} |x(n)|^2 = E.$$

JUSTIFY YOUR ANSWER!!!