

1. [10 pts.] If $x[n]$ has DTFT $X(\omega) = \frac{2e^{-j7\omega}}{1 - \frac{1}{2}e^{-j\omega}}$, find $x[n]$ for all n . **Show your work!**

2. A causal LTI system has transfer function $H(s) = \frac{s^2 + 1}{(s + 1)(s^2 - s - 6)}$.

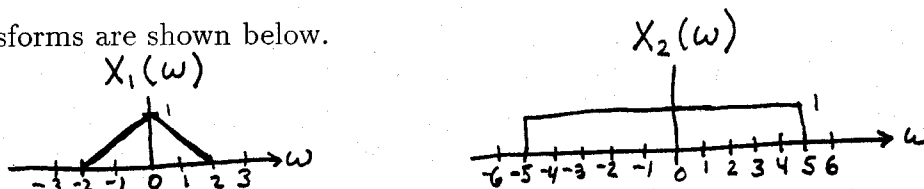
(a) [10 pts.] Sketch the pole-zero plot *and* indicate the region of convergence (ROC).

(b) [5 pts.] Is the system stable? **Give a reason for your answer!**

3. [10 pts.] Find $y(t)$ if it is a causal signal satisfying

$$\ddot{y}(t) + 7\dot{y}(t) + 12y(t) = 0; \quad y(0) = 11, \quad \dot{y}(0) = -13.$$

4. [15 pts.] Let $x_1(t)$ and $x_2(t)$ be a pair of bandlimited continuous-time waveforms whose Fourier transforms are shown below.



In order to reconstruct $y(t) = x_1(t)x_2(t)$ from samples $y(nT)$, what is the largest possible value of T according to the sampling theorem? **Justify your answer!**

5. [11 pts.] Consider the discrete-time LTI system defined by $y[n] = \sum_{k=0}^{\infty} \frac{x[n-k]}{\ln((2+k)^2)}$.

Is this system stable? **Justify your answer!** *Hint:* $\ln \theta < \theta$ for $\theta > 0$.

6. [12 pts.] Let $x(t) = \int_{-1}^1 \omega^3 \sin(\omega t) d\omega$. Evaluate $\int_{-\infty}^{\infty} |x(t)|^2 dt$. **Show your work!**

7. [12 pts.] The unit step $u(t)$ is applied to an LTI system, and the observed system output is $y(t) = \frac{1}{3}u(t) - \frac{1}{3}e^{-3t}u(t)$. Find the impulse response of the system, $h(t)$, for all t .

Simplify your answer as much as possible!

8. [15 pts.] Consider the continuous-time LTI system defined by $y(t) = \int_{-\infty}^{\infty} \frac{\sin(t-\tau)}{t-\tau} x(\tau) d\tau$.

Find $y(t)$ for all t if $x(\tau) = \frac{\sin(\tau/2)}{\tau}$.