Justify your answers! Be precise! Closed book. Closed Notes. No Calculators.

1. [20 pts.] Let X be a random variable with mean m and variance σ^2. Find a constant c that minimizes $\mathsf{E}[(X-c)^2]$. **Justify your answer!!!**
2. [20 pts.] The Bingy 007 jet aircraft's autopilot has conditional probability 1/3 of failure given that it employs a faulty Hexium III microprocessor chip. The autopilot has conditional probability 1/10 of failure given that it employs nonfaulty chip. According to the chip manufacturer, unrel, the probability of a customer's receiving a faulty Hexium III chip is 1/4. Given that an autopilot failure has occurred, find the conditional probability that a faulty chip was used.
3. [20 pts.] Professor Random has taught probability for many years. He has found that 80% of students who do the homework pass the exam, while 10% of students who don't do the homework pass the exam. If 60% of the students do the homework, what percent of students pass the exam? Of students who pass the exam, what percent did the homework?
4. [20 pts.] Consider a sequence of coin tosses modeled as follows. Let Ω be a sample space equipped with probability $\wp$. Let $A_1, A_2, \ldots$, be a sequence of *independent* events. Let A_i represent the event that the ith toss of the coin is "heads." Assuming that $\wp(A_i) = p$ for all $i = 1, 2, \ldots$, where $0 < p < 1$, find the probability that a "heads" occurs after the 100th toss. **JUSTIFY YOUR ANSWER!!!**
5. [10 pts.] Let X be a Poisson random variable with parameter λ. Let $Y = 1$ if $X \leq 1$ and $Y = 0$ if $X > 1$. Find $\mathsf{E}[Y]$.
6. [20 pts.] Let X be a nonnegative, integer-valued random variable with probability generating function $G_X(z) := \mathsf{E}[z^X] = (1-p)/(1-zp)$, where $0 < p < 1$. Find $\mathsf{E}[X]$, $\mathsf{E}[X^2]$, and $\mathsf{var}(X)$. *Hint:* Recall that $G'_X(1) = \mathsf{E}[X]$ and $G''_X(1) = \mathsf{E}[X(X-1)]$.
7. The university buys workstations from two different suppliers, Moon Microsystems (MM) and Hyped–Technology (HT). On delivery, 10% of MM's workstations are defective, while 20% of HT's workstations are defective. The university buys 140 MM workstations and 60 HT workstations.
   (a) [5 pts.] What is the probability that a workstation is from MM? From HT?
   (b) [10 pts.] What is the probability of a defective workstation?
   (c) [10 pts.] Given that a workstation is defective, what is the probability that it came from Moon Microsystems?
8. Let X and Y be discrete random variables. Suppose that given $Y = k$, X is conditionally Poisson with parameter k. If Y has mean m and variance r, find $\mathsf{E}[X^2]$.
9. Let $X \sim \text{binomial}(n, p)$, and let $Y \sim \text{binomial}(m, p)$. Put $Z = X + Y$. Assuming X and Y are independent, find the pmf of Z. Find $\wp(Z = 10 | X = 4)$ (assume $n \geq 4$ and $m \geq 6$).
10. Let X and Y be as in Problem 9 in Chapter 2. Find the probability generating function of Y. Then find $\wp(Y = 2)$.
11. Let X and Y be as in Example 2.4. Find the probability generating function of Y.