Justify your answers! Be precise! Closed book. Closed Notes. No Calculators.

1. [20 pts.] The Bingy 007 jet aircraft's autopilot has conditional probability 1/3 of failure given that it employs a faulty Hexium III microprocessor chip. The autopilot has conditional probability 1/10 of failure given that it employs nonfaulty chip. According to the chip manufacturer, unrel, the probability of a customer's receiving a faulty Hexium III chip is 1/4. Given that an autopilot failure has occurred, find the conditional probability that a faulty chip was used.
2. [10 pts.] Let X be a Poisson random variable with parameter λ. Let $Y = 1$ if $X \leq 1$ and $Y = 0$ if $X > 1$. Find $\mathsf{E}[Y]$.
3. [20 pts.] Let X be a nonnegative, integer-valued random variable with probability generating function $G_X(z) := \mathsf{E}[z^X] = (1-p)/(1-zp)$, where $0 < p < 1$. Find $\mathsf{E}[X]$, $\mathsf{E}[X^2]$, and $\mathsf{var}(X)$. *Hint:* Recall that $G'_X(1) = \mathsf{E}[X]$ and $G''_X(1) = \mathsf{E}[X(X-1)]$.
4. The university buys workstations from two different suppliers, Moon Microsystems (MM) and Hyped–Technology (HT). On delivery, 10% of MM's workstations are defective, while 20% of HT's workstations are defective. The university buys 140 MM workstations and 60 HT workstations.
   (a) [5 pts.] What is the probability that a workstation is from MM? From HT?
   (b) [10 pts.] What is the probability of a defective workstation?
   (c) [10 pts.] Given that a workstation is defective, what is the probability that it came from Moon Microsystems?
5. Let $X \sim \text{Poisson}(\lambda)$, and suppose that given $X = n$, $Y \sim \text{Bernoulli}(1/(n+1))$.
   (a) [20 pts.] Find $\wp(X = n|Y = 1)$.
   (b) [15 pts.] Find $\mathsf{E}[XY]$.
6. Let $X \sim \text{binomial}(n, p)$.
   (a) [15 pts.] Evaluate $\mathsf{var}(X)$.
   (b) [10 pts.] Evaluate $\wp(X < 2)$ if $n = 4$ and $p = 1/2$.
7. In Chapter 2, work Problems 33[15 pts.], 39–42.
8. Let $X \sim \text{geometric}_1(p)$. Show that its probability generating function is

$$\frac{(1-p)z}{1-zp}.$$