Poisson Process Problems

1. Hits to a certain web site occur according to a Poisson process of rate $\lambda = 3$ per minute. What is the probability that there are no hits in a 10-minute period? Give a formula and then evaluate it to obtain a numerical answer.
2. Cell-phone calls processed by a certain wireless base station arrive according to a Poisson process of rate $\lambda = 12$ per minute. What is the probability that more than 3 calls arrive in a 20-second interval? Give a formula and then evaluate it to obtain a numerical answer.
3. Let N_t be a Poisson process with rate $\lambda = 2$, and consider a fixed observation interval $(0, 5]$.
   (a) What is the probability that $N_5 = 10$?
   (b) What is the probability that $N_i - N_{i-1} = 2$ for all $i = 1, \ldots, 5$?
4. Let N_t be a Poisson process with rate λ.
   (a) Show that $N_{t+\Delta t} - N_t$ and N_t are independent.
   (b) Show that $\wp(N_{t+\Delta t} = k+\ell | N_t = k) = \wp(N_{t+\Delta t} - N_t = \ell)$.
   (c) Evaluate $\wp(N_t = k | N_{t+\Delta t} = k + \ell)$.
   (d) Show that as a function of $k = 0, \ldots, n$, $\wp(N_t = k | N_{t+\Delta t} = n)$ has the binomial(n, p) probability mass function and identify p.