Suppose that when \( A \) is a two-dimensional subset of the plane \( \mathbb{R}^2 \), we can compute \( \mathbb{P}(X,Y) \in A \).

Then how can we compute probabilities that involve only \( X \) or \( Y \), say \( \mathbb{P}(X \in B) \) or \( \mathbb{P}(Y \in C) \), where \( B \) and \( C \) are one-dimensional sets?

We can use Cartesian products:

\[
B \times C := \{(x,y) : x \in B \text{ and } y \in C\}
\]

IF \( B \& C \) ARE INTERVALS
THEN \( B \times C \) LOOKS LIKE:

In particular, \( [a,b] \times \mathbb{R} \) is the infinite vertical strip

and \( \mathbb{R} \times [c,d] \) is the infinite horizontal strip
We can now see that
\[
P(x \in B) = P(\{x \in B\} \cap \Omega) = P(\{x \in B\} \cap \{y \in \mathbb{R}\}) = P((x, y) \in B \times \mathbb{R})
\]
and
\[
P(y \in C) = P(\Omega \cap \{y \in C\}) = P(\{x \in \mathbb{R}\} \cap \{y \in C\}) = P((x, y) \in \mathbb{R} \times C).
\]

**Question 2** Now suppose that we can only compute probabilities of the form
\[
F_{xy}(x, y) := P(X \leq x, Y \leq y) = P(x \in (-\infty, x], y \in (-\infty, y]) = P((x, y) \in (-\infty, x] \times (-\infty, y])
\]

The product set \((-\infty, x] \times (-\infty, y]\) is the "southwest corner":

How can we compute the probability that \((X, Y)\) lies in a rectangle, say \([a, b] \times [c, d]\)?

We can find the answer by thinking of the rectangle as a part of a bigger southwest corner.
Let \( R := (a, b] \times (c, d] \) denote the desired rectangle. Let \( A_1 := (-\infty, a] \times (-\infty, d] \) denote the southwest corner.

Let \( A_2 := (-\infty, b] \times (-\infty, c] \) denote the southwest corner.

Then the big southwest corner \( (-\infty, b] \times (-\infty, d] \) is equal to \( R \cup A_1 \cup A_2 \) disjoint.
So,

\[ F_x(b,d) = P((x,y) \in (-\infty,b] \times (-\infty,d]) \]

\[ = P((x,y) \in \mathbb{R}) + P((x,y) \in A_1 \cup A_2) \]

Now, since \( A_1 \) and \( A_2 \) are not disjoint, but contain the south west corner \((-\infty,a]\times(-\infty,c]\) in common, we use the inclusion-exclusion formula to write

\[ P((x,y) \in A_1 \cup A_2) = P((x,y) \in A_1) + P((x,y) \in A_2) \]

\[ - P((x,y) \in (-\infty,a]\times(-\infty,c]) \]

\[ = F_{xy}(a,d) + F_{xy}(b,c) - F_{xy}(a,c). \]

So,

\[ P((x,y) \in \mathbb{R}) = F_{xy}(b,d) - F_{xy}(a,d) - F_{xy}(b,c) + F_{xy}(a,c), \]

the rectangle formula

\[ \begin{array}{ccc}
  & & \\
  & + & \\
  & & \\
 - & & \\
 + & & \\
 a & b & \\
\end{array} \]