

- (a) Show that the DTFT  $Y(f) = 1/(1 - ae^{j2\pi f})$  satisfies

$$|Y(f)|^2 = \frac{1}{1 - 2a\cos(2\pi f) + a^2}.$$

Observe that the denominator satisfies

$$(1 - a)^2 < 1 - 2a\cos(2\pi f) + a^2 < (1 + a)^2$$

and that  $(1 - a)^2 < 1$  and  $(1 + a)^2 > 1$ . The denominator has a maximum at  $f = \pm 1/2$  and a minimum at  $f = 0$ . Sketch the denominator and then sketch  $|Y(f)|^2$ . What happens as  $a \rightarrow 1$ ?

- (b) Show that the DTFT of  $y_n = a^{|n|}$  is

$$Y(f) = \frac{1 - a^2}{1 - 2a\cos(2\pi f) + a^2}$$

Sketch  $Y(f)$ .

- (c) Find the DTFT of  $y_n = a^n e^{j2\pi f_0 n}$  for  $n \geq 0$  and  $y_n = 0$  for  $n < 0$ .  
(d) Find the DTFT of  $y_n = a^{|n|} e^{j2\pi f_0 n}$ .