Continuous-Time Fourier Transform (CTFT)

$$X(f) \;=\; \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$

Inversion Formula

$$x(t) \;=\; \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$$

$x(t)$	$X(f)$
$I_{[-T,T]}(t)$	$2T\,\mathrm{sinc}(2Tf)$
$2f_c\,\mathrm{sinc}(2f_ct)$	$I_{[-f_c,f_c]}(f)$
$(1-\lvert t\rvert/T)I_{[-T,T]}(t)$	$T\,\mathrm{sinc}^2(Tf)$
$f_c\,\mathrm{sinc}^2(f_ct)$	$(1-\lvert f\rvert/f_c)I_{[-f_c,f_c]}(f)$
$e^{-\lambda t}u(t)$	$\dfrac{1}{\lambda+j2\pi f}$
$e^{-\lambda\lvert t\rvert}$	$\dfrac{2\lambda}{\lambda^2+(2\pi f)^2}$
$\dfrac{\lambda}{\lambda^2+t^2}$	$\pi e^{-2\pi\lambda\lvert f\rvert}$
$e^{-(t/\sigma)^2/2}$	$\sqrt{2\pi}\,\sigma e^{-\sigma^2(2\pi f)^2/2}$
$1/(\pi t)$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$(1/2)\delta(f)+1/(j2\pi f)$

Note. The indicator function $I_{[a,b]}(t) := 1$ for $a \le t \le b$ and $I_{[a,b]}(t) := 0$ otherwise. In particular, $u(t) := I_{[0,\infty)}(t)$ is the unit step function. Also, $\mathrm{sinc}(t) := [\sin(\pi t)]/(\pi t)$ for $t \neq 0$ and $\mathrm{sinc}(0) := 1$.

Series Formulas

$\displaystyle\sum_{k=0}^{N-1} z^k \;=\; \frac{1-z^N}{1-z}, \quad z \neq 1$	$\displaystyle e^z \;:=\; \sum_{k=0}^{\infty}\frac{z^k}{k!}$	$\displaystyle (a+b)^n \;=\; \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$
$\displaystyle\sum_{k=0}^{\infty} z^k \;=\; \frac{1}{1-z}, \quad \lvert z\rvert < 1$	$\displaystyle \lim_{n\to\infty}\Big(1+\frac{z}{n}\Big)^n \;=\; e^z$	$\displaystyle \sum_{k=1}^{n} k \;=\; \frac{n(n+1)}{2}$

Discrete-Time Fourier Transform (DTFT)

$$X(f) \;=\; \sum_{n=-\infty}^{\infty} x_n e^{-j2\pi fn}$$

Inversion Formula

$$x_n \;=\; \int_{-1/2}^{1/2} X(f)e^{j2\pi fn}\,df$$

x_n	$X(f)$	
$e^{j2\pi f_0 n}I_{[0,N-1]}(n)$	$N\dfrac{\mathrm{sinc}\big(N(f-f_0)\big)}{\mathrm{sinc}(f-f_0)}e^{-j\pi(f-f_0)(N-1)}$	
$2f_c\,\mathrm{sinc}(2f_cn)$	$I_{[-f_c,f_c]}(f)$,	$\lvert f\rvert \le 1/2,\ 0 < f_c \le 1/2$
$2f_2\,\mathrm{sinc}(2f_2n)-2f_1\,\mathrm{sinc}(2f_1n)$	$I_{[f_1,f_2]}(\lvert f\rvert)$,	$\lvert f\rvert \le 1/2,\ 0 \le f_1 < f_2 \le 1/2$
$e^{j2\pi f_0 n}$	$\displaystyle\sum_{k=-\infty}^{\infty}\delta(f-f_0-k)$,	(Dirac delta)
$a^nI_{[0,N-1]}(n)$	$\dfrac{1-(ae^{-j2\pi f})^N}{1-ae^{-j2\pi f}}$,	$\lvert a\rvert \neq 1$
$a^nu(n)$	$\dfrac{1}{1-ae^{-j2\pi f}}$,	$\lvert a\rvert < 1$
$a^{\lvert n\rvert}$	$\dfrac{1-a^2}{1-2a\cos(2\pi f)+a^2}$,	$\lvert a\rvert < 1$

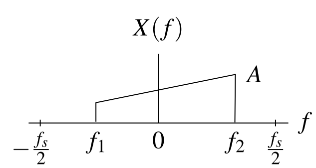
Relationship Between the CTFT and the DTFT

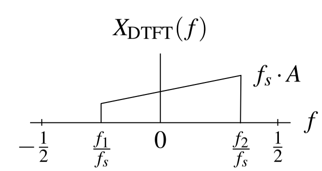
$$\frac{1}{f_s}X_{\mathrm{DTFT}}(f/f_s) = \sum_{n=-\infty}^{\infty} X(f-nf_s) =: \widetilde{X}(f) \qquad\qquad X_{\mathrm{DTFT}}(f) = f_s\widetilde{X}(f_sf).$$

If $x(t)$ is bandlimited to $f_c < f_s/2$, then $\widetilde{X}(f) = X(f)$ for $\lvert f\rvert \le f_s/2$,

$$x(t) = \sum_{m=-\infty}^{\infty} x(m/f_s)\,\mathrm{sinc}(f_s[t-m/f_s]),$$

and we have the graphical relationship





The z Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n}, \qquad \text{for } z \text{ such that } \sum_{n=-\infty}^{\infty} \left| x_n z^{-n} \right| < \infty.$$

x_n	$X(z)$	ROC
$a^n I_{[0,N-1]}(n)$	$\frac{1-(az^{-1})^N}{1-az^{-1}}$,	$z \neq a$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$,	$\|z\| > \|a\|$
$a^n u(-n)$	$\frac{1}{1-a^{-1}z}$,	$\|z\| < \|a\|$
$a^n u(-n-1)$	$\frac{-1}{1-az^{-1}}$,	$\|z\| < \|a\|$
$a^{\|n\|}$	$\frac{1}{1-az^{-1}} + \frac{az}{1-az}$,	$\|a\| < \|z\| < 1/\|a\|$