

Continuous-Time Fourier Transform (CTFT)

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

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Inversion Formula

$x(t)$	$X(f)$
$I_{[-T, T]}(t)$	$2T \operatorname{sinc}(2Tf)$
$2f_c \operatorname{sinc}(2f_c t)$	$I_{[-f_c, f_c]}(f)$
$(1 - t /T) I_{[-T, T]}(t)$	$T \operatorname{sinc}^2(Tf)$
$f_c \operatorname{sinc}^2(f_c t)$	$(1 - f /f_c) I_{[-f_c, f_c]}(f)$
$e^{-\lambda t} u(t)$	$\frac{1}{\lambda + j2\pi f}$
$e^{-\lambda t }$	$\frac{2\lambda}{\lambda^2 + (2\pi f)^2}$
$\frac{\lambda}{\lambda^2 + f^2}$	$\pi e^{-2\pi \lambda f }$
$e^{-(t/\sigma)^2/2}$	$\sqrt{2\pi} \sigma e^{-\sigma^2 (2\pi f)^2/2}$
$1/(\pi t)$	$-j \operatorname{sgn}(f)$
$u(t)$	$(1/2) \delta(f) + 1/(j2\pi f)$

Note. The indicator function $I_{[a,b]}(t) := 1$ for $a \leq t \leq b$ and $I_{[a,b]}(t) := 0$ otherwise. In particular, $u(t) := I_{[0, \infty)}(t)$ is the unit step function. Also, $\operatorname{sinc}(t) := [\sin(\pi t)]/(\pi t)$ for $t \neq 0$ and $\operatorname{sinc}(0) := 1$.

Series Formulas

$$\sum_{k=0}^{N-1} z^k = \frac{1-z^N}{1-z}, \quad z \neq 1 \quad \left| \quad e^z := \sum_{k=0}^{\infty} \frac{z^k}{k!} \quad \right| \quad (a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

$$\sum_{k=0}^{\infty} z^k = \frac{1}{1-z}, \quad |z| < 1 \quad \left| \quad \lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n = e^z \quad \right| \quad \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Discrete-Time Fourier Transform (DTFT)

$$X(f) = \sum_{n=-\infty}^{\infty} x_n e^{-j2\pi fn}$$

$$x_n = \int_{-1/2}^{1/2} X(f) e^{j2\pi fn} df$$

Inversion Formula

x_n	$X(f)$
$e^{j2\pi f_0 n} I_{[0, N-1]}(n)$	$N \frac{\operatorname{sinc}(N(f-f_0))}{\operatorname{sinc}(f-f_0)} e^{-j\pi(f-f_0)(N-1)}$
$2f_c \operatorname{sinc}(2f_c n)$	$I_{[-f_c, f_c]}(f), \quad f \leq 1/2, 0 < f_c \leq 1/2$
$2f_2 \operatorname{sinc}(2f_2 n) - 2f_1 \operatorname{sinc}(2f_1 n)$	$I_{[f_1, f_2]}(f), \quad f \leq 1/2, 0 \leq f_1 < f_2 \leq 1/2$
$e^{j2\pi f_0 n}$	$\sum_{k=-\infty}^{\infty} \delta(f-f_0-k), \quad (\text{Dirac delta})$
$a^n I_{[0, N-1]}(n)$	$\frac{1 - (ae^{-j2\pi f})^N}{1 - ae^{-j2\pi f}}, \quad a \neq 1$
$a^n u(n)$	$\frac{1}{1 - ae^{-j2\pi f}}, \quad a < 1$
$a^{ n }$	$\frac{1 - a^2}{1 - 2a \cos(2\pi f) + a^2}, \quad a < 1$

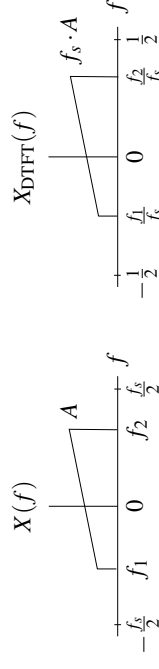
Relationship Between the CTFT and the DTFT

$$\frac{1}{f_s} X_{\text{DTFT}}(f/f_s) = \sum_{n=-\infty}^{\infty} X(f - nf_s) =: \tilde{X}(f) \quad X_{\text{DTFT}}(f) = f_s \tilde{X}(f_s f).$$

If $x(t)$ is bandlimited to $f_c < f_s/2$, then $\tilde{X}(f) = X(f)$ for $|f| \leq f_s/2$,

$$x(t) = \sum_{m=-\infty}^{\infty} x(m/f_s) \operatorname{sinc}(f_s[t - m/f_s]),$$

and we have the graphical relationship



The z Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n}, \quad \text{for } z \text{ such that } \sum_{n=-\infty}^{\infty} |x_n z^{-n}| < \infty.$$

x_n	$X(z)$	ROC
$a^n I_{[0, N-1]}(n)$	$\frac{1 - (az^{-1})^N}{1 - az^{-1}}$,	$z \neq a$
$a^n u(n)$	$\frac{1}{1 - az^{-1}}$,	$ z > a $
$a^n u(-n)$	$\frac{1}{1 - a^{-1}z}$,	$ z < a $
$a^n u(-n-1)$	$\frac{-1}{1 - az^{-1}}$,	$ z < a $
$a^{ n }$	$\frac{1}{1 - az^{-1}} + \frac{az}{1 - az}$,	$ a < z < 1/ a $