1. **MATLAB.** The MATLAB function `entropy2` is given below. Put it in an M-file and use the commands

\[
p = \begin{bmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix};
\]

\[
H = \text{entropy2}(p)
\]

to compute the entropy of \( p = (1/6, 1/3, 1/2) \). How does your answer compare with \( \log_2(3) \)?

```matlab
function H = entropy2(pin,varargin);

% H = entropy2(p,q) = -\sum_{i:q(i)>0} q(i)*\log_2(p(i))
% If q=p, use H = entropy2(p).
% p and q can be matrices; e.g.,
% 
% -\sum_{i,j} q(i,j)*\log p(j|i)
% 
% where q(i,j) may be the joint pmf.
% 
% You can also compute the relative entropy or
% informational divergence D(q||p) = entropy2(p./q,q)

if nargin > 1
    qin = varargin{1};
else
    qin = pin;
end
lt = prod(size(pin));
p = reshape(pin,lt,1); % convert pin to column vector
q = reshape(qin,1,lt); % convert qin to row vector
i = find(q>0);
H = - q(i) * log2( p(i) );
```

2. **MATLAB.** The MATLAB function `relent` is given below. Put it in an M-file and use the commands

\[
p = \begin{bmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix};
\]

\[
q = \begin{bmatrix} \frac{7}{10} & \frac{1}{5} & \frac{1}{10} \end{bmatrix};
\]

\[
D = \text{relent}(p,q)
\]

to compute \( D(p\|q) \), where \( p = (1/6, 1/3, 1/2) \) and \( q = (7/10, 1/5, 1/10) \).
function D = relent(p,q)
% Compute the relative entropy or Kullback-Leibler
% informational divergence sum_i p(i) log2(p(i)/q(i)).
D = entropy2(q./p,p);

3. **MATLAB.** When X and Y are finite sets, we can identify X with the positive integers \(\{1, \ldots, |X|\}\), and we can identify Y with the positive integers \(\{1, \ldots, |Y|\}\). Consider a joint pmf of the form \(P_{XY}(i,j) = p(i)W(j|i)\). We regard the input pmf \(p = [p(1), \ldots, p(|X|)]\) as a row vector. Since \(P_Y(j) = \sum_i P_{XY}(i,j) = \sum_i p(i)W(j|i)\), we can also regard \(P_Y\) as a row vector. In fact, if we let \(W\) be the matrix whose \(ij\) component is \(W(j|i)\), then in MATLAB, the expression \(PY=p*W\) computes the row vector of output probabilities \(P_Y\). The commands

\[
[m,n] = \text{size}(W); \quad \% m = \# \text{ inputs}, n = \# \text{ outputs} \\
Pmat = \text{repmat}(p',1,n); \quad \% Pmat \text{ is a matrix of inputs} \\
PXY = Pmat.*W;
\]

compute the matrix whose \(ij\) component is \(P_{XY}(i,j) = p(j)W(j|i)\). The commands

\[
PY = p*W \\
PYmat = \text{repmat}(PY,m,1); \\
PXY = \text{repmat}(PXY,m,1); \\
Pmat = \text{repmat}(p',1,n); \\
PXgY = PXY./PYmat;
\]

compute the matrix whose \(ij\) component is \(P_{X|Y}(i|j) = P_{XY}(i,j)/P_Y(j)\). Run

\[
W = [1/2 1/2 0; 1/4 0 3/4; 1/2 1/2 0]; \\
p = [5/12 1/3 1/4]
\]

\[
[m,n] = \text{size}(W); \\
Pmat = \text{repmat}(p',1,n); \\
PXY = Pmat.*W; \\
PY = p*W \\
PYmat = \text{repmat}(PY,m,1); \\
PXY = PXY./PYmat;
\]

\[
HX = \text{entropy2}(p) \\
HY = \text{entropy2}(PY) \\
HXY = \text{entropy2}(PXY) \\
HYgX = \text{entropy2}(W,PXY) \\
HXgY = \text{entropy2}(PXgY,PXY)
\]

to compute \(H(X), H(Y), H(XY), H(Y|X), \) and \(H(X|Y)\).

4. (a) Show that if \(X\) and \(Y\) have the same pmf, then \(H(Y|X) = H(X|Y)\).

(b) **MATLAB.** If \(P_X = P_Y\), is it true that \(P_{X|Y} = P_{Y|X}\)? Explore this question as follows. Recall Problem 3. In the script of that problem, \(P_Y = P_X\). Run
the script and print out $P_{XgY}$ and $W$. Are they the same?

(c) **MATLAB.** Compute $\text{relent}(P_{XY}, p' \ast PY)$ and compare it with the mutual information $H_Y - H_{YgX}$. 