ECE 729, Lec. 1
Exam 1
11 March 1997

100 Points

Justify your answers! **Be precise!**

Closed book. Closed Notes. No Calculators.

1. [15 pts.] Let $\mathsf{X} := \{1,2,3,4,5\}$. Suppose $\mathsf{P}(X=1)=\mathsf{P}(X=2)=1/4$, and $\mathsf{P}(X=3)=\mathsf{P}(X=4)=\mathsf{P}(X=5)=1/6$. Construct a binary Huffman code, and compute its expected length in bits.

2. [15 pts.] In the following variable-length binary code, z denotes either 0 or 1. The zs in different positions can be different. Is it possible to choose the zs so that the code is uniquely decodable? **Justify your answer!!!**

x	$C(x)$
a	z
b	zz
c	zz
d	zzz

3. [15 pts.] Let $p(x)$ be a fixed probability mass function (pmf) on X, and let $W(y|x)$ be a fixed transition pmf from X into Y. For arbitrary pmf $q(x)$ on X, put

$$K(q) := \sum_x \sum_y p(x) W(y|x) \log \frac{W(y|x)}{(qW)(y)},$$

where $(qW)(y) := \sum_x q(x)W(y|x)$. Show that $K(q) \geq K(p)$.

4. Which of the following are true/false? You do NOT need to justify your answer.
   (a) [5 pts.] $I(XZ \wedge Y) = I(X \wedge Y) + I(X \wedge Y|Z)$.
   (b) [5 pts.] $I(Y \wedge XZ) + I(X \wedge Z) = I(X \wedge YZ) + I(Y \wedge Z)$.
   (c) [5 pts.] $H(Y|XZ) = H(Y|X) + H(Z|XY) - H(Z|X)$.

5. Let f be a given function taking points x from a finite set X into points $y = f(x)$ in another finite set Y. Let X be an X-valued random variable with pmf p, and put $Y := f(X)$.
   (a) [10 pts.] Show that $I(X \wedge Y) = H(Y)$.
   (b) [15 pts.] Find a pmf p that achieves $\sup_p I(X \wedge Y)$.

6. [15 pts.] Let U and V be independent $\{0,1\}$-valued random variables with $\mathsf{P}(U=1)=\mathsf{P}(V=1)=0.9$. The pair UV is compressed according to the following block source code:

$U\ V$	X
0 0	0
0 1	0
1 0	0
1 1	1

and the result X is sent over a binary symmetric channel with crossover probability $\varepsilon = 0.01$. The channel output Y is uncompressed according to $0 \to 1\ 0$ and $1 \to 1\ 1$. If $\widehat{U}\widehat{V}$ is the result of uncompressing Y, find $\mathsf{P}(U\ V \neq \widehat{U}\ \widehat{V})$. You may assume that channel errors are independent of the source pair UV.