Weakly Symmetric Parallel Channels

Let $W_1(z_1|x)$ and $W_2(z_2|y)$ be weakly symmetric, put

$$P_{XY\underbrace{Z_1Z_2}_{=:Z}}(x,y,z_1,z_2) = p(x)q(y)\underbrace{W_1(z_1|x)W_2(z_2|y)}_{=:W(z_1,z_2|x,y)}.$$

Then

$$\begin{aligned} H_{12} &:= H(Z_1Z_2 \mid X=x, Y=y) \text{ does not depend on } x, y, \\ &= H(Z_1 \mid X=x) + H(Z_2 \mid Y=y) \\ &=: \quad H_1 \quad + \quad H_2 . \end{aligned}$$

Also, $Z_1 \perp\!\!\!\perp Z_2$.

Thus,

$$\begin{aligned} I(XY \wedge Z_1Z_2) &= H(Z_1Z_2) - H_1 - H_2 \\ &= H(Z_1) - H_1 + H(Z_2) - H_2. \end{aligned}$$

Since $P_{Z|Y}(z|y) = P_{Z_1}(z_1)W_2(z_2|y)$,

$$H(Z_1Z_2 \mid Y=y) = H(Z_1) + H_2 .$$

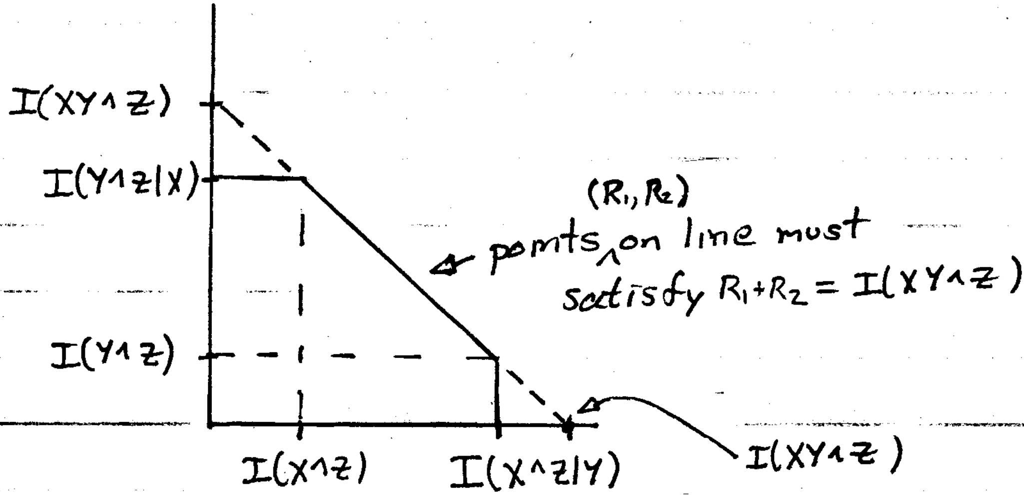
Thus,

$$\begin{aligned} I(X \wedge Z_1Z_2 \mid Y) &= H(Z|Y) - H(Z|XY) \\ &= \big(H(Z_1) + H_2\big) - (H_1 + H_2) \\ &= H(Z_1) - H_1 . \end{aligned}$$

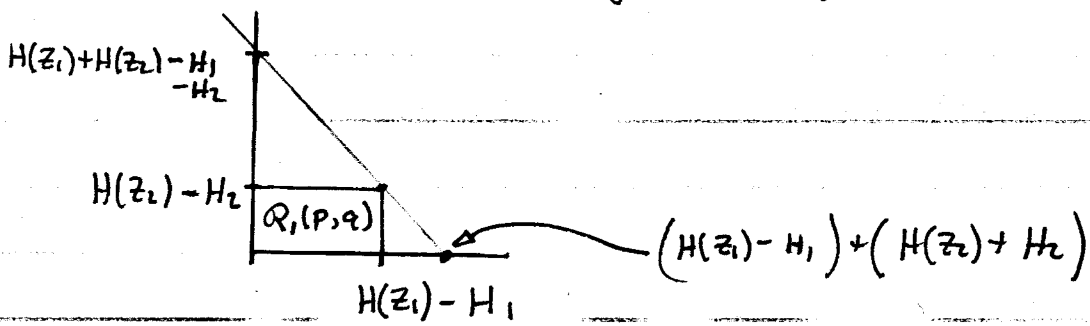
Similarly,

$$\begin{aligned} I(Y \wedge Z \mid X) &= H(Z_2) - H_2, \\ I(Y \wedge Z) &= H(Z_1) + H(Z_2) - [H(Z_1) + H_2] \\ &= H(Z_2) - H_2 \\ &= I(Y \wedge Z \mid X), \end{aligned}$$

and $I(X \wedge Z \mid Y) = I(X \wedge Z)$.



In this example, the pentagon is degenerate:



Largest rectangle is $[0, C(W_1)] \times [0, C(W_2)]$

since $H(Z_1)-H_1$ depends only on p
and $H(Z_2)-H_2$ depends only on q.