ECE 730

Modern Probability Theory and Stochastic Processes

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Class Webpage

What Will You Learn in 730?

- Probability Models
  - Random variables
    - The usual + gamma, Erlang, chi squared, Rayleigh Nakagami, Rice, noncentral chi squared
  - Random vectors – especially the Gaussian
  - Random processes
    - Markov chains
    - Poisson process
    - Wiener process

- Tools
  - Conditional probability and conditional expectation, the law of total probability, the smoothing property
  - Moment generating function, characteristic function, especially for independent random variables
  - Karhunen-Loeve expansion
  - Limits
    - To define new quantities; e.g., infinite sums and integrals of random processes
    - To approximate statistics; e.g., the central limit theorem and the law of large numbers:

$$\mathsf{P}(X_n \in B) \approx \mathsf{P}(X \in B) \qquad \mathsf{E}[X_k] \approx \frac{1}{n}\sum_{k=1}^{n} X_k$$

Course Organization

- Homework: assigned on Wed., due following Wed.
- Office Hours: TBA & I will almost always be available after every class to answer questions
- There will be a midterm night exam – TBA

Some Considerations

- Comm/DSP students
  - For a PhD, 730 is REQUIRED, and is covered on the PhD Qualifying Exam.
  - For a MS, 730 is NOT required.
- Other areas: 730 is NOT required.

1.2 Review of Set Notation

- Subset $A \subset B$
- Complement A^c (not compliment)
- Empty set or null set $\varnothing$
- Union $A \cup B$
- Intersection $A \cap B$
- Set difference $A \backslash B = A \cap B^c$
- Disjoint or mutually exclusive $A \cap B = \varnothing$

Set Identities

- Commutative: $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- Associative: $A \cup (B \cup C)=(A \cup B) \cup C$ and $A \cap (B \cap C)=(A \cap B) \cap C$
- Distributive: $A \cap (B \cup C)=(A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C)=(A \cup B) \cap (A \cup C)$
- De Morgan: $(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c=A^c \cap B^c$

Infinite Operations

- Generalized distributive laws

$$B \cap \left(\bigcup_{n=1}^{\infty} A_n \right) = \bigcup_{n=1}^{\infty} (B \cap A_n) \qquad B \cup \left(\bigcap_{n=1}^{\infty} A_n \right) = \bigcap_{n=1}^{\infty} (B \cup A_n)$$

- Generalized De Morgan's laws

$$\left(\bigcap_{n=1}^{\infty} A_n \right)^{\mathrm{c}} = \bigcup_{n=1}^{\infty} A_n^{\mathrm{c}} \qquad \left(\bigcup_{n=1}^{\infty} A_n \right)^{\mathrm{c}} = \bigcap_{n=1}^{\infty} A_n^{\mathrm{c}}$$

- Partition: Disjoint sets whose union is the whole space Ω.

Functions

- In the notation $f : X \rightarrow Y$ the set X is called the domain and the set Y is called the co-domain.
- The range of f is $\{ f(x) : x \in X \}$, which is a proper subset of Y.
- A function $f : X \rightarrow Y$ is 1-to-1 *if*

$$f(x_1) = f(x_2) \implies x_1 = x_2.$$

- A function $f : X \rightarrow Y$ is onto if

for every $y \in Y$, there is at least one $x \in X$ with $f(x) = y$.

Functions (continued)

- A function is onto <==> its range is equal to the co-domain.
- A function is invertible <==> it is both 1-to-1 and onto.
- Even if f is not invertible, if B is a subset of Y, we put

$$f^{-1}(B) := \{ x \in X : f(x) \in B \}$$