What Will You Learn in 730?

• Probability Models
  • Random variables
    – The usual + gamma, Erlang, chi squared, Rayleigh
      Nakagami, Rice, noncentral chi squared
  • Random vectors – especially the Gaussian
  • Random processes
    – Markov chains
    – Poisson process
    – Wiener process
• **Tools**
  
  • Conditional probability and conditional expectation, the law of total probability, the smoothing property
  
  • Moment generating function, characteristic function, especially for independent random variables
  
  • Karhunen-Loeve expansion
  
  • **Limits**
    
    – To define new quantities; e.g., infinite sums and integrals of random processes
    
    – To approximate statistics; e.g., the central limit theorem and the law of large numbers:

\[
P(X_n \in B) \approx P(X \in B) \quad \text{and} \quad E[X_k] \approx \frac{1}{n} \sum_{k=1}^{n} X_k
\]
Course Organization

- Homework: assigned on Wed., due following Wed.
- Office Hours: TBA & I will almost always be available after every class to answer questions
- There will be a midterm night exam – TBA
Some Considerations

- Comm/DSP students
  - For a PhD, 730 is REQUIRED, and is covered on the PhD Qualifying Exam.
  - For a MS, 730 is NOT required.
- Other areas: 730 is NOT required.
1.2 Review of Set Notation

• Subset $A \subset B$
• Complement $A^c$ (not compliment)
• Empty set or null set $\emptyset$
• Union $A \cup B$
• Intersection $A \cap B$
• Set difference $A \setminus B = A \cap B^c$
• Disjoint or mutually exclusive $A \cap B = \emptyset$
Set Identities

- Commutative: \(A \cup B = B \cup A\) and \(A \cap B = B \cap A\)

- Associative: \(A \cup (B \cup C) = (A \cup B) \cup C\) and \(A \cap (B \cap C) = (A \cap B) \cap C\)

- Distributive: \(A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\) and \(A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\)

- De Morgan: \((A \cap B)^c = A^c \cup B^c\) and \((A \cup B)^c = A^c \cap B^c\)
Infinite Operations

- Generalized distributive laws
  \[
  B \cap \left( \bigcup_{n=1}^{\infty} A_n \right) = \bigcup_{n=1}^{\infty} (B \cap A_n)
  \]
  \[
  B \cup \left( \bigcap_{n=1}^{\infty} A_n \right) = \bigcap_{n=1}^{\infty} (B \cup A_n)
  \]

- Generalized De Morgan's laws
  \[
  \left( \bigcap_{n=1}^{\infty} A_n \right)^c = \bigcup_{n=1}^{\infty} A_n^c
  \]
  \[
  \left( \bigcup_{n=1}^{\infty} A_n \right)^c = \bigcap_{n=1}^{\infty} A_n^c
  \]

- Partition: Disjoint sets whose union is the whole space \( \Omega \).
Functions

• In the notation $f : X \rightarrow Y$ the set $X$ is called the **domain** and the set $Y$ is called the **co-domain**.

• The **range** of $f$ is $\{ f(x) : x \in X \}$, which is a proper subset of $Y$.

• A function $f : X \rightarrow Y$ is **1-to-1** if $f(x_1) = f(x_2) \implies x_1 = x_2$.

• A function $f : X \rightarrow Y$ is **onto** if for every $y \in Y$, there is at least one $x \in X$ with $f(x) = y$. 
Functions (continued)

- A function is **onto** $\iff$ its range is equal to the co-domain.
- A function is **invertible** $\iff$ it is both **1-to-1** and **onto**.
- Even if $f$ is not invertible, if $B$ is a subset of $Y$, we put

$$f^{-1}(B) := \{ x \in X : f(x) \in B \}$$