ECE 730, Lec. 1
Exam 1
Tuesday, 21 Oct. 2003
5:15–6:45 pm

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in.× 11 in. paper on which you have prepared formulas.

1. [20 pts.] Internet packets arrive at a router according to a Poisson process of rate λ. Find the variance of the time it takes for the first n packets to arrive.

2. [20 pts.] Let W_t be a Wiener process with $\mathsf{E}[W_t^2] = \sigma^2 t$. Put $Y_t := e^{W_t}$. Find the correlation function $R_Y(t_1, t_2) := \mathsf{E}[Y_{t_1} Y_{t_2}]$ for $t_2 > t_1$.

3. [20 pts.] Let $X_1, \ldots, X_n$ be random variables, and define

$$Y_k := \sum_{i=1}^{k} X_i, \quad k = 1, \ldots, n.$$

   Suppose that $Y_1, \ldots, Y_n$ are jointly Gaussian. Are $X_1, \ldots, X_n$ jointly Gaussian? **Justify your answer.**

4. [20 pts.] Let X and Y be zero-mean random vectors with C_Y and C_{XY} given. Do *not* assume C_Y is invertible. Let $\widehat{X} = AY$ be the linear MMSE estimator of X based on Y. The error covariance is defined to be $\mathsf{E}[(X - \widehat{X})(X - \widehat{X})']$. Of the following formulas

   (a) $C_X - AC_{YX}$

   (b) $C_X - C_{XY}A'$

   (c) $C_X - AC_{YX} - C_{XY}A' + AC_YA'$

   list all the ones, if any, that are valid expressions for the error covariance. **Show your work.**

5. [20 pts.] Let Ω be an uncountable set. Let $\mathcal{A}$ denote the collection of all subsets A such that either A is countable or A^c is countable. Is $\mathcal{A}$ a σ-field? **Justify your answer.**