ECE 730, Lec. 1
Exam 1
Tuesday, 19 Oct. 2004
7:15–8:45 pm

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in.$\times$ 11 in. paper on which you have prepared formulas.

1. [15 pts.] Let X and Y be positive random variables with joint density $f_{XY}(x,y)$. Put

$$U := XY \quad \text{and} \quad V := X/Y.$$

   Find the joint density of U and V.

2. [20 pts.] Let V be an Erlang random variable with parameters $m = 2$ and $\lambda = 1$. Let $U \sim \text{uniform}[-1/2, 1/2]$. Put $Y := e^{VU}$, and find the density $f_Y(y)$ for all y, assuming that V and U are independent.

3. [25 pts.] Let $X = [X_1, \ldots, X_n]'$ be a random vector with zero mean and covariance matrix C_X. Put $Y := [X_1, \ldots, X_m]'$, where $m < n$. Find the linear MMSE estimator of X based on Y. Your answer should be in terms of the block components of C_X,

$$C_X = \begin{bmatrix} C_1 & C_2 \\ C_2' & C_3 \end{bmatrix},$$

   where C_1 is $m \times m$ and invertible.

4. [20 pts.] Let $X_1, \ldots, X_n$ be i.i.d. $N(0,1)$ random variables. Find the density of

$$Y := (X_1 + \cdots + X_n)^2.$$

5. [20 pts.] Let Ω be a nonempty set, and let $\mathcal{F}$ and $\mathcal{G}$ be σ-fields. Is $\mathcal{F} \cup \mathcal{G}$ a σ-field? If "yes," prove it. If "no," give a counterexample.