ECE 730, Lec. 1
Exam 1
Wednesday, 26 Oct. 2005
5:15–6:45 pm
Room 376 Mechanical Engineering

100 Points

Justify your answers!　　　　　**Be precise!**

Closed Book　　　　　**Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. [10 pts.] Let $\mathcal{H}$ be a σ-field of $\mathbb{R}$ that contains all sets of the form $(a,b]$, where $-\infty < a < b < \infty$. Determine whether or not $\mathcal{H}$ contains all sets of the form $(-\infty,b]$. **Justify your answer**.

2. [25 pts.] Let X and Z be random variables such that Z is observed and an estimate of X is to be constructed. Consider estimates of the form

$$\sum_{k=1}^{n} a_k Z^k,$$

   where $a_1, \ldots, a_n$ are to be determined to minimize

$$\mathsf{E}\left[\left|X - \sum_{k=1}^{n} a_k Z^k\right|^2\right].$$

   Find equations that the optimal a_k must satisfy.

3. [25 pts.] The time to send an internet packet is $T \sim \exp(\lambda)$, and the time to receive the acknowledgement is $A \sim \exp(\mu)$. If T and A are independent, find $\mathsf{P}(T \le z/2 | T + A = z)$. Evaluate all integrals.

4. [20 pts.] Let X and Z be independent with $X \sim N(0,1)$ and $\mathsf{P}(Z = \pm 1) = 1/2$. Determine whether or not $Y := Z\sqrt{X^2}$ is Gaussian. **Justify your answer.**

5. [20 pts.] Let X be a random variable, and put $A := \{x \in \mathbb{R} : \mathsf{P}(X = x) > 0\}$. Show that A is a countable set. Do not assume that X is a discrete random variable. *Hint:* You may use the fact that the set $A_n := \{x \in \mathbb{R} : \mathsf{P}(X = x) > 1/n\}$ contains at most n points.