ECE 730
Exam 1
30 March 2009
5:30–7:00 pm in 3534 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. [20 pts.] Let Ω denote the positive integers. Let $\mathscr{A}$ denote the collection of all subsets A such that either A is finite or A^c is finite. Let $\mathscr{B}$ denote the smallest σ-algebra that contains $\mathscr{A}$. Determine whether or not $\mathscr{B}$ contains all subsets of Ω. **Justify your answer.**

2. [20 pts.] Let X be a positive random variable with mean m and variance σ^2. Given $X = x$, suppose $\{N_t, t \geq 0\}$ is a Poisson process with rate x. Find the linear MMSE estimate of X based on observing N_t at a fixed time t.

3. [20 pts.] Let $X_1, \ldots, X_n$ be jointly Gaussian random variables. Let X_0 be another random variable such that for all $c_1, \ldots, c_n$,

$$X_0 + \sum_{k=1}^{n} c_k X_k$$

is a scalar Gaussian random variable. Determine whether or not $X_0, X_1, \ldots, X_n$ are jointly Gaussian. **Justify your answer.**

4. [20 pts.] Let $X \sim N(0, \sigma_X^2)$ and $Y \sim N(0, \sigma_Y^2)$. Put

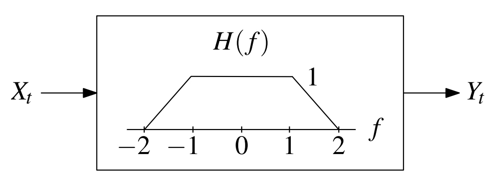
$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}.$$

If X and Y are independent, is it possible to choose *nonzero* a, b, c, d so that U and V are independent? **Justify your answer.**

5. [20 pts.] Let X_t be a zero-mean, wide-sense stationary random process with power-spectral density

$$S_X(f) = \begin{cases} 1 - f^2, & |f| \leq 1, \\ 0, & |f| > 1. \end{cases}$$

Suppose X_t is applied to the linear, time-invariant system with transfer function $H(f)$ shown below



Evaluate $\mathsf{E}[|Y_t - X_t|^2]$. **Show your work.**