

ECE 730
Final Exam
11 May 2009
10:05 am – 12:05 pm in 3534 EH

100 Points

Justify your answers!

Be precise!

Closed Book

Closed Notes

**You may bring two sheets of 8.5 in. × 11 in. paper
on which you have prepared formulas.**

1. [15 pts.] Let X and Y be jointly Gaussian random variables with $X \sim N(0, \sigma_X^2)$, $Y \sim N(0, \sigma_Y^2)$, and with X and Y having correlation coefficient

$$\rho := \frac{E[XY]}{\sigma_X \sigma_Y}.$$

- (a) Find $E[X|Y]$.
- (b) Find $E[|X - E[X|Y]|^2]$.
2. [10 pts.] Let $\{N_t, t \geq 0\}$ be a Poisson process with intensity λ . For $0 < s < t$, find $E[N_t|N_s]$.
3. [15 pts.] Let U_1, U_2, \dots be i.i.d. uniform $[0, 1]$ random variables. Let $N \sim \text{Poisson}(\lambda)$ be independent of $\{U_k\}_{k=1}^\infty$. Find the probability density of $Z := \max(U_1, \dots, U_{N+1})$.
4. [20 pts.] Let $\{W_t, t \geq 0\}$ be a Wiener process, and put

$$X_n := \sum_{k=1}^n W_{k-1}(W_k - W_{k-1}), \quad n \geq 1.$$

Determine whether or not $\{X_n, n \geq 1\}$ is a martingale with respect to $\{W_n, n \geq 1\}$. **Justify your answer.**

5. [20 pts.] Let X_n converge in probability to X . Put $Y_n := \sin(X_n)$ and $Y := \sin(X)$. Determine whether or not Y_n converges in mean of order one to Y . **Justify your answer.**
6. [20 pts.] Suppose $X_n \sim \text{Cauchy}(\lambda_n)$ and converges almost surely to a random variable X . Determine whether or not X is a Cauchy random variable. **Justify your answer.**