

**ECE 730**  
**Exam 1**  
**21 March 2011**  
**5:30–7:00 pm in 3534 EH**

**100 Points**

**Justify your answers!**

**Be precise!**

**Closed Book**

**Closed Notes**

**You may bring one sheet of 8.5 in. × 11 in. paper  
on which you have prepared formulas.**

- [20 pts.] Let  $X$  have density  $f(x) = \exp[x - e^x]$  for  $-\infty < x < \infty$ . Find the moment generating function of  $X$ . **Evaluate all integrals. Show your work!**
- [20 pts.] Consider the sample space  $\Omega := \{1, 2, 3, 4\}$  equipped with the  $\sigma$ -algebra

$$\mathcal{A} := \{\emptyset, \{1, 2\}, \{3, 4\}, \Omega\}$$

and probability measure

$$P(A) := \begin{cases} 0, & A = \emptyset, \\ 1/3, & A = \{1, 2\}, \\ 2/3, & A = \{3, 4\}, \\ 1, & A = \Omega. \end{cases}$$

If

$$X(\omega) := I_{\{1,2\}}(\omega) - \omega I_{\{3,4\}}(\omega), \quad \omega \in \Omega,$$

determine whether or not  $E[X]$  is well defined. **Justify your answer.**

- [20 pts.] Let  $X$  and  $Y$  have joint density

$$f_{XY}(x, y) = y^2 e^{-y^2 x}, \quad x \geq 0 \text{ and } 1 \leq y \leq 2.$$

Compute  $E[X^3 Y^2]$ . **Evaluate all integrals. Show your work!**

- [20 pts.] Let  $X$  be an  $n$ -dimensional Gaussian random vector with zero mean and covariance matrix  $C$ . Find the moment generating function of  $\|X\|^2$ . *Hint:* Your answer should be in terms of the eigenvalues of  $C$ . **Evaluate all expectations/integrals.**
- [20 pts.] The amount of snow that falls during a snow storm is an  $\exp(\lambda)$  random variable. In a series of  $n$  independent snow storms, find the probability that  $k$  of them drop more than  $t$  inches of snow. **Show your work/explain your analysis.**