ECE 730
Exam 1
25 October 2012
5:00–6:30 pm in 2255 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. Let $\Omega := \{1,2,3,4\}$, and consider the function

$$X(\omega) := I_{\{1,4\}}(\omega) + \omega I_{\{2,3\}}(\omega),$$

which takes the values 1, 2, and 3. If there is a suitable σ-algebra of Ω on which a probability measure P is defined, then we can write

$$\mathsf{E}[X] = \mathsf{P}(\{1,4\}) + 2\mathsf{P}(\{2\}) + 3\mathsf{P}(\{3\}).$$

Of course, the σ-algebra of all subsets of Ω will work. Is there is a smaller σ-algebra will work? **Justify your answer — explain your reasoning.**

2. Let Y be an exponential random variable with parameter one, and given $Y = y$, suppose X is conditionally Cauchy(y). Compute $\mathsf{E}[Y^n \cos(X)]$. **Evaluate all integrals.**

3. Let $X = [X_1, \ldots, X_n]'$ be Gaussian random vector with zero mean and nonsingular covariance matrix C whose ij entry is denoted by C_{ij}. Let $b = [b_1, \ldots, b_n]'$ be a deterministic, nonzero vector, and put $Y := b'X$. Compute $\mathsf{E}[X_1 | Y = y]$. Your answer should be in terms of y, C, and b (or the entries C_{ij} and b_j). **Explain your reasoning; justify your analysis.**

4. Let X and Y be zero-mean random vectors with covariance matrices C_X, C_Y, and C_{XY}. Let A and B be deterministic matrices that satisfy

$$AC_Y = C_{XY} \quad \text{and} \quad BC_Y = C_{XY}.$$

If C_Y is singular, is $\mathsf{E}[\|AY - BY\|^2] = 0$? **Justify your answer.**

5. A new digital energy detector for radio transmissions takes two independent samples X and Y and triggers an alarm if the total energy $X^2 + Y^2$ exceeds a given threshold t. If X and Y are both $N(0,1)$, find the probability that the alarm is triggered. **Evaluate all integrals.**