ECE 730
Final Exam
21 December 2012
5:05–7:05 pm in 2317 EH

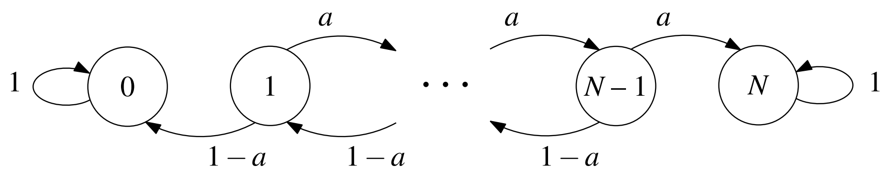
100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring two sheets of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. Consider the discrete-time Markov chain with state transition diagram below:



   where $0 \le a \le 1$. Are there any values of a for which X_n is a martingale with respect to itself? **Justify your answer.**

2. Let X and Y be random variables with $X \in L^1$. Let q be an invertible function, and put $Z := q(Y)$. Put $\widehat{g}(y) := \mathsf{E}[X|Y = y]$. Determine whether or not $\mathsf{E}[X|Z] = \widehat{g}(q^{-1}(Z))$. **Justify your answer.**

3. Let X be a zero-mean Gaussian random vector with invertible covariance matrix C. For $t > 0$, put
$$B_t := \{x : x'C^{-1}x > t\}.$$
   If X has dimension $2n$, find a simple formula (no integrals) for $\mathsf{P}(X \in B_t)$.

4. Let N_t be a Poisson process with intensity λ. Put $Y_t := g(N_t)$, where
$$g(x) := \begin{cases} x, & 0 \le x < 1, \\ 1 + x, & x \ge 1. \end{cases}$$
   For $0 \le s < t < \infty$, compute $\mathsf{E}[g(N_t) - g(N_s)]$.

5. Let m_n be an arbitrary sequence of real numbers, and let σ_n be an arbitrary sequence of positive numbers. Let X be a Laplace random variable with parameter $\lambda = 1$. Define a sequence of random variables $Y_n := \sigma_n X + m_n$. Assume Y_n converges in mean of order 2 to some random variable Y. Determine whether or not Y is a continuous random variable. **Justify your answer.**