ECE 730
Exam 1
21 October 2013
5:15–6:45 pm in 3345 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. Your friend is learning about probability and is working with the probability space $(\Omega, \mathscr{A}, \mathsf{P})$, where the sample space is $\Omega := \{a,b,c,d\}$, the sigma-algebra is $\mathscr{A} := \big\{\varnothing, \{a,b\}, \{c,d\}, \Omega\big\}$, and the probability measure is given by $\mathsf{P}(\{a,b\}) := 1/3$, and $\mathsf{P}(\{c,d\}) := 2/3$. Determine whether or not it is possible to construct a random variable X on this probability space that satisfies $\mathsf{P}(X=1) = 1/3$, $\mathsf{P}(X=2) = 1/3$, and $\mathsf{P}(X=3) = 1/3$. **Justify your answer.**

2. Let $X \sim N(0,1)$ and $N \sim \text{Poisson}(\lambda)$ be independent random variables. Evaluate

$$\mathsf{E}\bigg[\int_0^X t^N\,dt\bigg].$$

   For full credit, simplify your answer as much as possible.

3. Let $X_0, X_1, \ldots, X_n$ be random variables applied as input to the moving-average filter with coefficients $h_0, \ldots, h_n$. The system output is

$$Y_i := \sum_{k=0}^{i} h_k X_{i-k}, \quad i = 0, \ldots, n.$$

   Assume $Y_0, \ldots, Y_n$ are jointly Gaussian. Find conditions on the filter coefficients that force $X_0, \ldots, X_n$ to be jointly Gaussian. **Justify your answer.**

4. Let X be a random vector with zero-mean and nonsingular covariance matrix C_X. Put $Y := MX$, where M is a given full-rank $m \times n$ matrix with $m \le n$. Find the *linear* MMSE estimator of X based on Y. Express your answer in terms of M, C_X, and Y.

5. Let X and Z be independent random variables with $X \sim \exp(\lambda)$ and $\mathsf{P}(Z = \pm 1) = 1/2$. Find the density of $Y := ZX$. **For full credit, simplify your answer as much as possible.**