ECE 730
Exam 1
27 October 2014
5:15–6:45 pm in 3534 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. Let X and W be independent random variables with zero means and unit variances. If $Y := \beta X + \sigma W$, find the **linear** MMSE estimate of X based on Y. Your answer should be an explicit formula in terms of β, σ, and Y. **Justify your answer.**

2. Let $\Omega := (-\infty, \infty)$, and let $\mathscr{A}$ denote the collection of all subsets of Ω of the form $(a,b]$, $(-\infty,a]$, and (b,∞) for all a and b with $-\infty < a < b < \infty$. Let $\mathscr{A}$ also include Ω and the empty set $\varnothing$. Determine whether or not $\mathscr{A}$ is a σ-algebra. **Justify your answer.**

3. On Campus Drive, the speed limit is 40 mph, and vehicle speeds have probability density

$$f(x) := \begin{cases} \frac{1}{5} e^{-(x-35)/5}, & x \geq 35, \\ 0, & x < 35. \end{cases}$$

   A police officer issues a ticket if a vehicle is going faster than 47 mph. If 10 vehicles pass the police officer, find the variance of the number of tickets issued. **State any additional assumptions you need to compute your answer. Justify the steps of your calculation.**

4. Let X and Y be independent random variables with $X \sim \text{Erlang}(m = 2, \lambda = 1)$ and $Y \sim \text{uniform}[0, 1/2]$. Simplify

$$\mathsf{E}\left[\int_0^Y e^{tX} \, dt \right].$$

   Your answer should have NO integrals. **Justify the steps of your calculation.**

5. Let X and Y be jointly Gaussian random vectors. Let $g(y) := \mathsf{E}[X|Y = y]$. Determine whether or not the error $X - g(Y)$ is a Gaussian random vector. **Justify your answer.**