ECE 730
Exam 1
21 October 2015
5:15–6:30 pm in 2540 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes**

You may bring one sheet of 8.5 in. $\times$ 11 in. paper on which you have prepared formulas.

1. Let $Y \sim N(0,\sigma^2)$, and given $Y = y$, let $X \sim \exp(y^2)$. Find $\mathsf{E}[X^2Y^6]$. **Evaluate all integrals.**

2. There are n students in a classroom, and each student has a random number of pencils X_i, where the X_i are i.i.d. uniformly distributed on $\{0,1,2,3,4,5,6,7,8,9\}$. For fixed k in the range $0,\dots,n$, find the probability that exactly k students have three or more pencils.

3. Let U and Y be zero-mean random vectors having given covariance matrices C_Y and C_{UY}. Let A solve $AC_Y = C_{UY}$, where C_Y is *not* assumed to be invertible. Find the *linear* MMSE estimate of $X := [U'\ Y']'$ based on Y. **Justify your answer.**

4. Consider random variables $U = X+Y$ and $V = X-Y$. If U and V are jointly Gaussian, determine whether or not X and Y are jointly Gaussian. **Justify your answer.**

5. Let Ω be a nonempty set, and let $\mathscr{A}$ be a σ-algebra of subsets of Ω (but not the collection of *all* subsets of Ω). Fix any set $B \subset \Omega$, where $B \notin \mathscr{A}$. Put $\mathscr{C} := \{A\cap B : A \in \mathscr{A}\}$. Determine whether or not $\mathscr{C}$ is a σ-algebra $\underline{\underline{\text{of } B}}$. *Hint:* To address this question, it is essential to take complements of subsets of B $\underline{\underline{\text{relative to } B}}$. In other words, if $D \subset B$, then the $\underline{\text{complement of } D \text{ relative to } B}$ is $D^{\mathrm{c}} \cap B$.