ECE 730, Lec. 1
Exam 1
Wednesday, 17 Oct. 2018
5:00 pm – 6:30 pm
2534 EH

100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes** **No Calculators**

You may bring one sheet of 8.5 × 11 paper with notes written on both sides.

The solutions of a quadratic equation $as^2 + bs + c = 0$ are

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1. [20 pts.] Let X and Y be random vectors, and suppose you are given

$$\begin{matrix} m_X := \mathsf{E}[X] & R_X := \mathsf{E}[XX'] \\ m_Y := \mathsf{E}[Y] & R_Y := \mathsf{E}[YY'] \end{matrix} \quad \text{and} \quad R_{XY} := \mathsf{E}[XY'].$$

   Express the **linear** MMSE estimate of X based on Y using any of the above quantities as appropriate. If you need the inverse of a matrix, assume it exists.

2. [20 pts.] Let $X \sim \exp(\lambda)$, and given $X = x$, let the conditional density of Y be $N(x^3, \sigma^2)$. Compute $\mathsf{E}[Y^2]$. **Show your work**.

3. [20 pts.] Let $Y = (X+W)^2$, where $X \sim N(0, 0.9)$, $W \sim N(0, 0.1)$, and X and W are independent. Find the density of Y. **Justify your answer.**

4. [20 pts.] If $X \sim \exp(\lambda)$ and $Y \sim \exp(\mu)$ are independent, find the density of $Z := \max(X, Y)$. **Show your work.**

5. [20 pts.] Suppose X is a two-dimensional random vector with zero mean and covariance matrix

$$C_X = \begin{bmatrix} \varepsilon & 1 \\ 1 & \varepsilon \end{bmatrix},$$

   where $\varepsilon > 1$ is a parameter. Let $X = PY$ be the Karhunen–Loève expansion of X. Find the covariance matrix of Y. Express your answer in terms of ε. *Hint:* This does not require finding the transformation P.