# ECE 730, Lec. 1
# Final Exam
# Tuesday, 18 Dec. 2018
# 2:45 pm – 4:45 pm
# 3444 EH

## 100 Points

**Justify your answers!**          **Be precise!**

**Closed Book**          **Closed Notes**          **No Calculators**

**You may bring two sheets of 8.5 × 11 paper with notes written on both sides.**

1. [15 pts.] Consider a linear, time-invariant system with impulse response $h(t) = e^{-t/\tau}u(t)$, where the time constant $\tau$ can lie in the range $0 < a \le \tau \le b < \infty$. Let $Y_t$ denote the system response to a zero-mean, wide-sense stationary, white-noise process $X_t$ with power spectral density $S_X(f) = N_0/2$. Determine the value of the time constant $\tau$ that minimizes the expected instantaneous output power $E[Y_t^2]$. **Justify your answer.**

2. [20 pts.] Let $X_n$ be a time-homogeneous Markov chain with transition probabilities $p_{ij}$ and initial distribution $v_i := P(X_0 = i)$. Determine whether or not

$$P(X_0 = i | X_1 = j, X_2 = k, X_3 = \ell) = P(X_0 = i | X_1 = j).$$

**Justify your answer.**

3. Suppose $\{N_t, t \ge 0\}$ is a Poisson process, and $E[N_6] = \mu$.

   (a) [10 pts.] Express $P(N_2 \le 2)$ directly in terms of $\mu$.

   (b) [15 pts.] Given $0 < s < t$, find the conditional probability generating function, $E[z^{N_t} | N_s]$.

4. Suppose $X_n$ converges in probability to $X$. Put $Y_n := X_n e^{-X_n}$ and $Y := Xe^{-X}$.

   (a) [10 pts.] Does $Y_n$ converge in probability to $Y$? **Briefly explain why or why not.**

   (b) [10 pts.] Does $E[Y_n] \to E[Y]$? **Briefly explain why or why not.**

5. [20 pts.] Let $X_i$ be a discrete-time, Gaussian random process, and consider the new process,

$$Y_k := \sum_{i=-\infty}^{\infty} h(k-i)X_i,$$

where $h(\cdot)$ is a given, deterministic, finite-duration signal. (Because $h(\cdot)$ is finite duration, for each fixed $k$, the above sum contains only finitely many terms, which means we do not have to worry about limits.) Determine whether or not $Y_k$ is a Gaussian random process. **Justify your answer.**