ECE 730, Lec. 1
Exam 1
Monday, 21 Oct. 2019
4:14 pm – 5:45 pm
2540 EH

100 Points

Justify your answers!   **Be precise!**

Closed Book   **Closed Notes**   **No Calculators**

You may bring one sheet of 8.5 × 11 paper with notes written on both sides.

PART 1 — Straightforward Application of Tools We've Studied

1. [20 pts.] Let $X \sim \text{gamma}(p, 1)$, and suppose that given $X = x$, Y is conditionally $\exp(x)$. Also assume that given $X = x$ and $Y = y$, Z is conditionally $N(0, y^2)$. Find $\mathsf{E}[X^4 Y Z^2]$.

2. [20 pts.] A new bridge has 4 cables. Let X_i denote the force on the ith cable. A cable will fail if the force on it exceeds t. If at most one cable fails, the bridge will remain standing.[1]

   (a) Write an expression for the event that the bridge remains standing.

   (b) Assuming the X_i are i.i.d. $\exp(\lambda)$ random variables, find a formula for the probability that the bridge remains standing.

3. [20 pts.] Suppose $Z = X + Y$, where X and Y are independent $\exp(\lambda)$ random variables. Find a formula for the conditional density $f_{Y|Z}(y|z)$ **and** specify the range of values of y and z where your density positive.

PART 2 — More Abstract

4. [20 pts.] Let $u_1, \ldots, u_n$ be an orthonormal basis for $\mathbb{R}^n$, and let Z be a zero mean random variable with variance σ^2. Put $X := Z u_1$. Find the "ingredients" of the Karhunen–Loève expansion of the random vector X; i.e., find a diagonal matrix Λ and a matrix P such $P' \text{cov}(X) P = \Lambda$ and $P'P = I$. **Justify your answer.**

5. [20 pts.] Consider the problem of estimating a random variable X based on repeated noisy measurements $Y_i = X + V_i$. Suppose that $X, V_1, \ldots, V_n$ are uncorrelated random variables, with the V_i all having mean zero and variance σ_V^2, and with X having mean m_X and variance σ_X^2. Since X is a scalar, the linear MMSE estimate of X based on the vector $Y := [Y_1, \ldots, Y_n]'$ has the form $A(Y - m_Y) + m_X$ where A is a **row vector**, say $A = [A_1, \ldots, A_n]$. Find A.

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[1]This was not the case for the Morandi bridge, which collapsed when the first cable stay failed.
See `https://www.pbs.org/wgbh/nova/video/why-bridges-collapse/?linkId=74764116`