ECE 730, Lec. 1
Final Exam
Monday, 16 Dec. 2019
12:25 pm – 2:25 pm
2540 EH

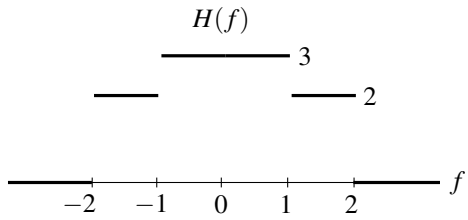
100 Points

Justify your answers! **Be precise!**

Closed Book **Closed Notes** **No Calculators**

You may bring two sheets of 8.5×11 paper with notes written on both sides.

1. [15 pts] Suppose $X \sim \exp(\lambda)$ and $Y \sim \exp(\mu)$, where X and Y are independent. Compute $\mathsf{E}[(X+Y)^2]$.
2. [15 pts] White noise with power spectral density $S_X(f) = N_0/2$ is applied to the lowpass filter $H(f)$ shown below.



   If the system output is denoted by Y_t, find the expected instantaneous output power $\mathsf{E}[Y_t^2]$.
3. [15 pts] Let X_n converge in probability to X, where $X \sim \text{Laplace}(\lambda)$.
   (a) Determine whether or not

$$\cos(X_n) \text{ converges in probability to } \cos(X).$$

   Justify your answer.

   (b) Determine whether or not

$$\lim_{n\to\infty} \mathsf{E}[\cos(X_n)] = \mathsf{E}[\cos(X)].$$

   Justify your answer.

   (c) Evaluate $\mathsf{E}[\cos(X)]$. *Hint:* Don't compute any integrals.
4. [15 pts.] Give an example of a Gaussian random vector $[X,Y]'$ that does *not* have a joint density, but for which at least one component does have a marginal density.
5. [20 pts.] Let $U \sim \text{uniform}[-2,2]$, and put $X_n := (4-U^2)^n$. Let $G := \{X_n \to 0\}$.
   (a) Compute $\mathsf{P}(G)$.
   (b) Does X_n converge almost surely to 0? **Justify your answer.**
6. [20 pts.] Suppose X_n converges in probability to X. Suppose also that B is a positive, finite constant such that $|X_n| \le B$ and $|X| \le B$. Determine whether or not X_n converges in mean of order 2 to X. **Justify your answer.**