ECE 735

HW # 12

1) Let Z be an inner-product space, and let X be a vector space. Let $A: X \to Z$ be linear. Fix any $z \in Z$. Define

$$f(x) := \mathrm{Re}\langle Ax, z\rangle$$

Fix any $\Delta x \in X$. <u>By direct calculation,</u> show that

$$\lim_{\lambda \downarrow 0} \frac{f(x+\lambda \Delta x) - f(x)}{\lambda} = \mathrm{Re}\langle A\Delta x, z\rangle$$

2) Let X be a real or complex inner-product space, and let $B: X \to X$ be self adjoint and positive semidefinite. Consider the real-valued function $f(x) := \langle Bx, x\rangle$. <u>By direct calculation</u>, show that

$$\lim_{\lambda \downarrow 0} \frac{f(x+\lambda \Delta x) - f(x)}{\lambda} = 2\mathrm{Re}\langle \Delta x, Bx\rangle.$$

3) Let X_0 be an arbitrary subset of an arbitrary set X, and let f be a real-valued function on X_0. Let Z be a real or complex inner-product space, and let $G: X_0 \to Z$. Put $L(\mu, x) := f(x) + \mathrm{Re}\langle \mu, G(x)\rangle$

for all $\mu \in Z$ and $x \in \mathfrak{X}_0$. Show that if there exists a $\mu_0 \in Z$ and an $x_0 \in \mathfrak{X}_0$ such that $G(x_0) = 0$ and

$$L(\mu_0, x_0) \le L(\mu_0, x), \quad \forall x \in \mathfrak{X}_0,$$

then

$$f(x_0) \le f(x) \quad \forall x \in \mathfrak{X}_0 \text{ with } G(x) = 0,$$

4) If in Problem 3, $\mathfrak{X}$ is a vector space and $G(x) = Ax - b$, where $A : \mathfrak{X} \to Z$ is linear and $b \in Z$, and if $\mathfrak{X}_0$ is convex and f is a convex function, show that $L(\mu, x)$ is a convex function of $x \in \mathfrak{X}_0$. Also compute $(D_x^+ L)(\mu, x, \Delta x)$ if $\mathfrak{X}_0 = \mathfrak{X}$ and $f(x) = \|x\|^2$, assuming $\mathfrak{X}$ is a real or complex inner-product space.