ECE 735, Fall 2008
Final Exam Review Questions

1) Consider the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) := e^{-x}$.
   a) What value of $x \in [0,1]$ minimizes f?
   b) Justify your answer (use a Lagrange Multiplier Theorem).

2) In Problem 7-19, find the adjoint of A. Also, can you find the adjoint of B?

3) Let $A: \mathbb{X} \to \mathbb{Y}$ and $B: \mathbb{Y} \to \mathbb{X}$ be linear operators on vector spaces $\mathbb{X}$ and $\mathbb{Y}$.
   (a) Show $BA = I \Rightarrow A$ is nonsingular
   (b) If $BA = I$ and A is onto, show that $AB = I$
   (c) If $\dim \mathbb{X} = \dim \mathbb{Y} < \infty$ and $BA = I$, show that $AB = I$.

4) Let a_n be a periodic sequence with period N. Define the $N \times N$ matrix A to have entries $A_{mk} := a_{k-m}$ for $k, m = 0, \dots, N-1$. Thus,

$$A = \begin{bmatrix} a_0 & a_1 & \cdots & a_{N-1} \\ a_{N-1} & a_0 & a_1 \cdots & a_{N-2} \\ a_{N-2} & a_{N-1} & a_0 \cdots & a_{N-3} \\ & \vdots & & \\ a_1 & \cdot & \cdot & a_0 \end{bmatrix}$$

Such a matrix is said to be <u>circulant</u>. Show that the eigenvalues of A are given by the DFT of the sequence a_n, and that the corresponding orthonormal eigenvectors are the columns of the DFT matrix U given by $U_{k\ell} := e^{-j2\pi k\ell/N}/\sqrt{N}$, $k,\ell = 0,\ldots,N-1$. More specifically, show that $U^H U = I$ and that $U^H A U = \mathrm{diag}(\hat{a}(0),\ldots,\hat{a}(N-1))$, where $\hat{a}(k) := \sum_{n=0}^{N-1} a_n e^{-j2\pi kn/N}$.

Note: Circulant matrices are a special kind of Toeplitz matrix. See Prof. Robert Gray's review for more info. (Link on our webpage) Only if you want more info.

5) Find $(x,y) \in \mathbb{R}^2$ to minimize $ax - by$ subject to $x^2 + y^2 \le r^2$, for given a, b, r. Prove you have found the global minimum.

6) Find $x > 0$, $y > 0$, $z > 0$ to maximize xyz subject to $xy + xz + yz = b$ for given $b > 0$.

Hint: minimize $-xyz$.