ECE 901

HW #2

1) For a Poisson process of rate $\lambda(\tau)$ and arrival times $T_1 < T_2 < \cdots$, put $Z := \sum_{k=1}^{N_t} g(T_k)$.

(a) Show that $E[Z^2] = \left[\int_0^t g(\tau)\lambda(\tau)\,d\tau\right]^2 + \int_0^t g(\tau)^2\lambda(\tau)\,d\tau$

(b) Show that $\mathrm{var}(Z) = \int_0^t g(\tau)^2\lambda(\tau)\,d\tau$.

(c) Show that

$$E[e^{sZ}] = \exp\left\{\int_0^t \left[e^{sg(\tau)} - 1\right]\lambda(\tau)\,d\tau\right\}$$

2) Let N_t be a homogeneous Poisson process with rate $\lambda = 1$. Let $r(t) > 0$, and put $R(t) := \int_0^t r(\tau)\,d\tau$. Then $R(t)$ is strictly increasing. Define a new process by

$$M_t := N_{R(t)}.$$

Show that M_t is a Poisson process with rate $r(t)$.

3) Write a Matlab function T=genpp(Tmax, lambda) that generates $T = [T_1 \cdots T_n]$ from a homogeneous Poisson process of rate lambda, where $0 < T_1 < \cdots < T_n \le T_{max} < T_{n+1}$

Use your function in Problem 3 to

4) Write a Matlab script to plot

$$\frac{1}{K}\sum_{k=1}^{K}\left(\sum_{i=1}^{n_k} h(t-T_{k,i})\right)$$

to approximate $E[\sum_{i=1}^{N_t} h(t-T_k)]$,

where $[T_{k,1} \cdots T_{k,n_k}]$ is the kth result of calling genpp.

<u>Hint:</u> Given t=linspace(0, 2*Tmax, 200); use the Matlab function lincmb(t, ones(size(T)), 'hfun', T) to compute

$$\sum_{i=1}^{n_k} h(t-T_{k,i})$$

where you can download lincmb.m from the class web page. ▨

Write your own Matlab function y=hfun(t) to compute

$$h(t) := \begin{cases} e^{-t}, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

Run your script with Tmax=10, lambda=1, and K=50.