ECE 901

HW #3

1) Let $Z(t)$ be AWGN. Let $\varphi_1, \ldots, \varphi_N$ be orthonormal. Put

$$\hat{Z} := \sum_{k=1}^{N} \langle Z, \varphi_k \rangle \varphi_k.$$

Compute $E[Z(t)\hat{Z}(\tau)^*]$ and $E[\hat{Z}(t)\hat{Z}(\tau)^*]$ and show that $E[(Z(t)-\hat{Z}(t))\hat{Z}(\tau)^*]=0$ for all t, τ.

2) Let X and Y be n-dimensional, jointly Gaussian, zero mean, and uncorrelated; i.e., $C_{XY} = C_{YX} = 0$. Also assume $C_X = C_Y = C$. Then

$$f_{XY}(x,y) = \frac{\exp\left\{-\frac{1}{2}[x' \ y'] \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \end{bmatrix}\right\}}{(2\pi)^{(2n)/2}\sqrt{(\det C)^2}},$$

since $\begin{bmatrix} x \\ y \end{bmatrix}$ is a $2n$-dim. Gaussian vector with covariance matrix $\begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}$ of size $2n \times 2n$.

Show that $f(x,y)$ is equal to the formula at the bottom of p. G-5.

3) Derive Craig's formula (first derive ⊛ on p. G-9 following the approach suggested there).