ECE 901
HW #4

1) <u>Optimality of the MAP decision rule.</u>

Let $p_i > 0$, $\sum_{i=0}^{M-1} p_i = 1$, $f_i(y) \geq 0$, $\int_{\mathbb{R}^d} f_i(y)\,dy = 1$,
and suppose

$$y \in D_i^* \Rightarrow p_i f_i(y) \geq p_j f_j(y) \text{ for all } j.$$

Let $D_0, \ldots, D_{M-1}$ be any partition of $\mathbb{R}^d$; i.e., $D_i \cap D_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=0}^{M-1} D_i = \mathbb{R}^d$. If $\psi^*(y) = i \Leftrightarrow y \in D_i^*$ and $\psi(y) = i \Leftrightarrow y \in D_i$, show that

$$\mathcal{P}(\psi^*(Y) = H) \geq \mathcal{P}(\psi(Y) = H).$$

<u>Hint</u>: Use the formula derived on p. D-2.

2) Use MATLAB to plot $P_T(\eta)$ based on Chebyshev-Gauss quadrature. Use $M(\theta) = \exp(\Lambda[e^\theta - 1])$ with $\Lambda = 10$. For $\eta_{dB} \in [-20, 10]$, put $\eta = 10^{\eta_{dB}/10}$. Plot η_{dB} and $P_T(\eta)$ on a semilog-Y scale.

Next, on the same graph plot

$$\frac{1}{M} \sum_{m=1}^{M} Q\left(\sqrt{2\eta N_m}\right)$$

where $N_1, \ldots, N_M$ are iid Poisson($\Lambda = 10$) RVs; use only 7 values of η_{dB} equally spaced in $[-20, 10]$. Use $M = 3000$.