ECE 901

HW #6

1) Let μ be a measure on a space $\mathbb{X}$, and let $f(x) \geq 0$. Define

$$\nu(B) := \int_B f(x)\,\mu(dx), \quad B \subset \mathbb{X}.$$

It can be shown that ν is also a measure on $\mathbb{X}$. Let $g(x) \geq 0$. Use LMCT to show that

$$\int g(x)\,\nu(dx) = \int g(x) f(x)\,\mu(dx).$$

2) Let N be a Poisson random counting measure on $\mathbb{X}$, and put

$$\Phi := \int_{\mathbb{X}} h\,dN$$

for some $h \geq 0$. If Λ is the mean measure of N, show that

$$E[Q(\sqrt{2\eta\Phi})] \geq \tfrac{1}{2} e^{-\Lambda(\mathbb{X})}, \quad \eta \geq 0.$$

3) Let $N_1, N_2, \ldots$ be $\perp\!\!\!\perp$ RVs, with $N_k \sim \text{Poisson}(\beta_k)$, $0 < \beta_k < \infty$. Put $M_\ell := \sum_{k=1}^{\ell} N_k$. (a) Show that $M_\ell \sim \text{Poisson}(\sum_{k=1}^{\ell} \beta_k)$.

(b) If $\sum_{k=1}^{\ell} \beta_k \to \beta < \infty$, show that M_ℓ converges in distribution to a Poisson(β) RV. (c) Show that $M := \lim_{\ell\to\infty} M_\ell < \infty$ with probability one by using LMCT to show that $E[M] = \lim_{\ell\to\infty} E[M_\ell] < \infty$.