1) Let \( \mu \) be a measure on a space \( \mathcal{X} \), and let \( f(x) \geq 0 \). Define

\[
\nu(B) := \int_B f(x) \mu(dx), \quad B \subset \mathcal{X}.
\]

It can be shown that \( \nu \) is also a measure on \( \mathcal{X} \). Let \( g(x) \geq 0 \). Use LMCT to show that

\[
\int g(x) \, \nu(dx) = \int g(x) f(x) \mu(dx),
\]

2) Let \( N \) be a Poisson random counting measure on \( \mathcal{X} \), and put

\[
\Phi := \int h \, dN
\]

for some \( h \geq 0 \). If \( \Lambda \) is the mean measure of \( N \), show that

\[
\mathbb{E} \left[ Q(\sqrt{2 \gamma \Phi}) \right] \geq \frac{1}{2} e^{-\Lambda(\mathcal{X})}, \quad \gamma \geq 0.
\]

3) Let \( N_1, N_2, \ldots \) be i.i.d. RVs, with \( N_k \sim \text{Poisson}(\beta_k) \), \( 0 < \beta_k < \infty \), \( \sum_k \beta_k < \infty \). Put \( M_k := \sum_{i=1}^k N_i \). Show that \( M_k \sim \text{Poisson}(\sum_{i=1}^k \beta_i) \).

5) If \( \frac{1}{k} \sum_{i=1}^k \beta_i \to \beta < \infty \), show that \( M_k \) converges in distribution to a \( \text{Poisson}(\beta) \) RV. Show that \( M := \lim_{k \to \infty} M_k < \infty \) with probability one by using LMCT to show that \( \mathbb{E}[M] = \lim_{k \to \infty} \mathbb{E}[M_k] < \infty \).