ECE 901

HW #7

1) For nonnegative numbers $x_{nm} \geq 0$, show directly that

$$\sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} x_{nm}\right) = \sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty} x_{nm}\right).$$

2) For $x \geq 0$, show that $0 \leq 1-e^{-x} \leq \min(x,1)$.
Hint: It suffices to establish the separate inequalities $0 \leq 1-e^{-x} \leq 1$ and $0 \leq 1-e^{-x} \leq x$.
Note that $1-e^{-x} = \int_{-x}^{0} e^{t}\,dt$.

3) For $g(x) \geq 0$, show that

$$0 \leq \int\left[1-e^{-g(x)}\right]\Lambda(dx) \leq \int \min(g(x),1)\,\Lambda(dx).$$

4) For $g(x) \geq 0$, put $G_+ := \{x : g(x) > 0\}$. Show that

$$\int \min(g(x),1)\,\Lambda(dx) \leq \Lambda(G_+).$$

5) Use Problem 1 to show that if $\mu_1, \mu_2, \ldots$ are measures on $\mathbb{X}$, then so is μ defined by

$$\mu(B) := \sum_{m=1}^{\infty} \mu_m(B).$$

6) With μ_m and μ as in Problem 5, show that for $g \geq 0$,

$$\int g\,d\mu = \sum_{m=1}^{\infty} \int g\,d\mu_m.$$

Hint: Use LMCT.