ECE 901
HW #8

1) <u>Def.</u> If X is a RV with MGF $M(s) := E[e^{sX}]$, then the <u>cumulant generating function (CGF)</u> of X is $K(s) := \ln E[e^{sX}] = \ln M(s)$.

<u>Problem:</u> Show that K is a convex function of s. <u>Hint:</u> Use the Cauchy-Schwarz inequality, which says that for nonnegative U, V, $E[UV] \le E[U^p]^{1/p} E[V]^{1/q}$ if $p, q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that $K(\frac{1}{p}s_1 + \frac{1}{q}s_2) \le \frac{1}{p}K(s_1) + \frac{1}{q}K(s_2)$.

2) Let N be a point process on $\mathbb{X}$ with Laplace functional $L_N(g) = E[e^{-\int g\,dN}]$ for functions $g(x) \ge 0$. Let $X_1, X_2, \ldots$ be the points of N, and let $Y_1, Y_2, \ldots$ be corresponding marks in some space $\mathbb{Y}$. The marked point process is

$$N^*(C) := \sum_{i=1}^{\infty} I_C(X_i, Y_i).$$

Recall that the marks are conditionally independent given the X_i's, and that Y_i given all X_j depends only on X_i. Thus, $E[v(Y_i) \mid X_1 = x_1, X_2 = x_2, \ldots] = E[v(Y_i) \mid X_i = x_i]$ for any function $v(y)$. Now, for any function $h(x,y) \ge 0$ define the conditional cumulant generating functional

$$K_h(x) := \ln E[e^{-h(x,Y_i)} \mid X_i = x] \le 0.$$

Show that $L_{N^*}(h) := E[e^{-\int h\,dN^*}]$ satisfies

$$L_{N^*}(h) = L_N(-K_h).$$