1) Def: If $X$ is a RV with MGF $M(s) := E[e^{sX}]$, then the **cumulant generating function (CGF)** of $X$ is $K(s) := \ln E[e^{sX}] = \ln M(s)$.

**Problem:** Show that $K$ is a convex function of $s$. **Hint:** Use the Cauchy-Schwarz inequality, which says that for nonnegative $U, V$,
\[ E[UV] \leq E[U^p]^{1/p} E[V^q]^{1/q} \quad \text{if} \quad p, q \geq 1 \quad \text{and} \quad \frac{1}{p} + \frac{1}{q} = 1. \]
Show that $K(q^{1/p}s_1 + q^{1/q}s_2) \leq \frac{q}{p} K(s_1) + \frac{p}{q} K(s_2)$.

2) Let $N$ be a point process on $\mathbb{X}$ with Laplace functional $L_N(g) := E[e^{-g \cdot dN}]$ for functions $g(x) \geq 0$. Let $X_1, X_2, \ldots$ be the points of $N$, and let $Y_1, Y_2, \ldots$ be corresponding marks in some space $\mathbb{Y}$. The marked point process is
\[ N^*(C) := \sum_{i=1}^{\infty} I_C(X_i, Y_i). \]

Recall that the marks are conditionally independent given the $X_i$'s, and that $Y_i$ given all $X_i$ depends only on $X_i$. Thus,
\[ E[\nu(Y_i) \mid X_1 = x_1, X_2 = x_2, \ldots] = E[\nu(Y_i) \mid X_i = x_i] \]
for any function $\nu(y)$. Now, for any function $h(x, y) \geq 0$ define the conditional cumulant generating functional
\[ K_h(x) := \ln E[ e^{-h(x, Y_i)} \mid X_i = x ] \leq 0. \]
Show that $L_N(h) := E[ e^{-h \cdot dN^*} ]$ satisfies
\[ L_N(h) = L_N(-K_h). \]