Discrete Wavelet Transforms for Images. We now briefly sketch a simple extension of one-dimensional wavelet transforms to handle two-dimensional images; i.e., where before x_0 was an $N \times 1$ vector, we now suppose that x_0 is an $N \times \tilde{N}$ matrix, where N and $\tilde{N}$ are both powers of 2. Let G and H be $N/2 \times N$ matrices that satisfy (4)–(7). Similarly, let $\tilde{G}$ and $\tilde{H}$ be $\tilde{N}/2 \times \tilde{N}$ matrices that also satisfy (4)–(7). Set

$$T \triangleq \begin{bmatrix} G \\ H \end{bmatrix} \quad \text{and} \quad \tilde{T} \triangleq \begin{bmatrix} \tilde{G} \\ \tilde{H} \end{bmatrix}.$$

By our assumptions on G, H and $\tilde{G}$, $\tilde{H}$, we have $T^*T = I$, $TT^* = I$, $\tilde{T}^*\tilde{T} = I$, and $\tilde{T}\tilde{T}^* = I$. Thus, the mapping $x_0 \to Tx_0\tilde{T}^*$ is a unitary operator.

Now observe that

$$Tx_0\tilde{T}^* = \begin{bmatrix} Gx_0\tilde{G}^* & Gx_0\tilde{H}^* \\ Hx_0\tilde{G}^* & Hx_0\tilde{H}^* \end{bmatrix}.$$

Let $x_1 = Gx_0\tilde{G}^*$ and let v_1 denote the remaining blocks of the above matrix. It should now be clear how to write the Decomposition and Reconstruction Algorithms using a sequence of matrices G_j, H_j, $\tilde{G}_j$, and $\tilde{H}_j$ that satisfy (4)–(7). If these matrices are obtained as in Section 3.2, then we require that $2 \leq p, \tilde{p} \leq \min\{N, \tilde{N}\}/2^J$.