ECE 430 HW on Sets

In each of the following problems, g is a real-valued function defined on a subset X of the real line. In each problem, (a) sketch the graph of the function g on its domain X; (b) indicate the set U on the graph, and (c) give a simplified analytical expression for U.

1. Let $X = [0, \infty)$, and for $x \in X$, $g(x) = x^2$. Find $U = \{x \in X : 4 \leq g(x) \leq 16\}$.
2. Same as 1 but $X = (-\infty, \infty)$.
3. Same as 1 but $X = [-3, \infty)$.
4. Let $X = (-\infty, \infty)$, and for $x \in X$, $g(x) = \cos(x)$.
   Find $U = \{x \in X : 1/2 \leq g(x) \leq 1\}$.
5. Let $X = (-\infty, \infty)$, and for $x \in X$, $g(x) = \begin{cases} 0, & |x| < 1, \\ |x| - 1, & 1 \leq |x| \leq 2, \\ 1, & |x| > 2. \end{cases}$
   Find $U = \{x \in X : g(x) \leq 1\}$.
6. Same as 5 but $U = \{x \in X : g(x) \leq 1/2\}$.
7. Same as 5 but $U = \{x \in X : g(x) \leq 0\}$.
8. Let $X = (-\infty, \infty)$, and for $x \in X$, $g(x) = \begin{cases} -x - 2, & x < -1, \\ -x^2, & -1 \leq x < 0, \\ x^3, & 0 \leq x < 1, \\ 1, & x \geq 1. \end{cases}$
   Find $U = \{x \in X : g(x) \leq 1\}$.
9. Same as 8 but $U = \{x \in X : g(x) \leq 1/8\}$.
10. Same as 8 but $U = \{x \in X : g(x) \leq 0\}$.
11. Same as 8 but $U = \{x \in X : g(x) \leq -1/4\}$.