Analysis of the IEEE 802.15.3a UWB Channel Model

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***Abstract*— The IEEE 802.15.3a standards body has developed a modification of the Saleh–Valenzuela (SV) multipath channel model as the accepted channel model for ultra-wideband (UWB) investigations. The SV model is a well-defined simulation model that is straightforward to implement. However, since the model as specified is not directly amenable to theoretical analysis, this paper develops it as a two-dimensional "augmented cluster point process" that is composed of three statistically independent components that can be separately analyzed. For the IEEE 802.15.3a model, simple closed-form expressions in terms of the channel parameters are derived for: 1) The mean and variance of the number of multipath arrivals in a time window as well as the covariance of the numbers of arrivals in nonoverlapping windows. 2) The mean and variance of the sum of the path gains in a time window; it is shown that the sums of gains in nonoverlapping windows are uncorrelated. 3) The expected energy of a received UWB waveform. These are all special cases of general integral formulas that are derived.**

***Index Terms*— Augmented cluster process, point process, Saleh–Valenzuela model, ultra-wideband.**

I. Introduction

Ultra-wideband (UWB) communication systems have recently generated intense interest due to their potential for providing pervasive wireless connectivity [9], [12]. This potential is due to the fact that UWB can provide very high bit rate, low-cost, low-power wireless communication for a wide variety of systems; e.g., personal computer, TV, VCR, CD, DVD, MP3 [1], [9]. Current systems, such as those based on IEEE 802.11b, 11a, or 11g cannot do this because their power consumption and cost are too high [1].

The Federal Communications Commission recently allocated 7.5 GHz of spectrum for unlicensed commercial ultra-wideband (UWB) communication systems. In order to develop a common channel model, the IEEE 802.15.3a standards body considered several possibilities and established a modification of the Saleh–Valenzuela (SV) model [10] as the accepted channel model for UWB investigations [1], [7]. The SV model is a well-defined simulation model that is straightforward to implement. Unfortunately, the usual specification of the SV model is not directly amenable to theoretical analysis.

To address this difficulty, in this paper we develop the SV/IEEE 802.15.3a model as a two-dimensional "augmented cluster point process" that is composed of three statistically independent components that can be separately analyzed.

In Section II we briefly introduce multipath channel models and show that they can be thought of as two-dimensional point processes. We then show that the response of such a channel to a waveform can be viewed as a shot-noise process driven by the two-dimensional point process.

Since the SV/IEEE 802.15.3a channel model is a kind of cluster point process, in Section III we give a brief introduction to such processes. We also introduce augmented cluster processes. We then foreshadow the construction of the SV/IEEE 802.15.3a process to come in Section IV and show how the process will decompose into three statistically independent components.

The power of this framework is illustrated in Section V. There we first give novel closed-form expressions for the mean and variance of the number of multipath arrivals in a time window. We also give the covariance of the number of arrivals in nonoverlapping time windows; note that since the IEEE 802.15.3a model is not a Poisson process, arrivals in disjoint time windows are not independent. As a more complicated example, we give novel closed-form expressions for the mean and variance of the sum of the gains of paths arriving in a specified time window. We also show that the sums of the gains in nonoverlapping time windows are uncorrelated under the IEEE 802.15.3a model. As the third example, we give novel expressions for the covariance function of a received UWB waveform under the IEEE 802.15.3a model. In particular, we give a simple closed-form expression for the expected energy in a received UWB waveform.

II. Multipath Channels, Point Processes, and Shot Noise

Frequency-selective fading channels are well modeled by time-varying impulse responses of the form [8]

$$h_p(t,\tau) \;=\; \sum_{l=1}^{L_p(t)} \beta_l(t)\delta(\tau-\tau_l(t)),$$

where t and τ are the observation time and the application time of the impulse, respectively. The total number of multipath components is $L_p(t)$, the $\{\beta_l\}$ are time-varying gains, and the $\{\tau_l(t)\}$ are the path arrival times or delays. In this paper we focus on indoor environments whose structure changes slowly in comparison with the signaling rate. This suggests that we use the corresponding time-invariant model

$$h_p(\tau) \;=\; \sum_{l=1}^{L_p} \beta_l\delta(\tau-\tau_l).$$

A. The Point-Process Connection

We can regard the pairs (τ_l,β_l) as random points in two-dimensional space as shown in Fig. 1. In other words, we regard the pairs (τ_l,β_l) as being points of a *two-dimensional*

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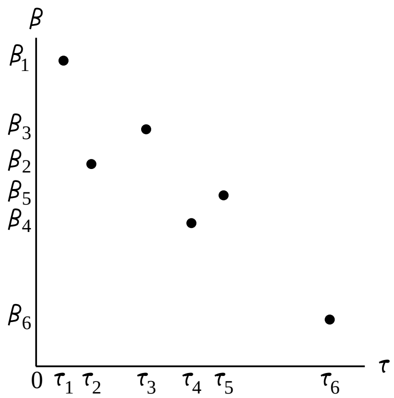

Fig. 1. Delay-gain pairs (τ_l, β_l) regarded as a two-dimensional point process.

point process. By definition, the delays τ_l are nondecreasing. However, the corresponding gains depend in part on the different scatterers encountered by the radio waves. Hence, although the general trend is to decrease, it is not monotonic.

Most prior work, e.g., [2], [3], [5], [10], [13], has always regarded the delays τ_l as a temporal point process and the β_l as marks. Although, the two perspectives are equivalent [4], [6], we find the abstract multidimensional viewpoint much more amenable to analysis.

B. The Shot-Noise Connection

The response $\rho(t)$ of the channel $h_p(\tau)$ to a signal $\xi(t)$ is

$$\rho(t) \;=\; \sum_{l=1}^{L_p} \beta_l \xi(t-\tau_l). \tag{1}$$

Observe that if we put $\varphi_t(\tau,\beta) := \beta\xi(t-\tau)$, then

$$\rho(t) \;=\; \sum_{l=1}^{L_p} \varphi_t(\tau_l,\beta_l).$$

Since for each t, $\rho(t)$ is a sum of values of a function evaluated at random points, $\rho(t)$ is called a shot-noise random variable. A collection of such random variables indexed by t is called a shot-noise process. This is an important observation because so much theory is available to analyze shot-noise (or filtered point processes), e.g., [4], [6], [11] to cite just a few.

C. The Clustering Phenomenon

We have now established that a multipath channel can be modeled as a point process in a two-dimensional space, and that its response to an input waveform has the structure of a shot-noise process. What remains is to specify the distribution of the times τ_l and the gains β_l. For example, Turin *et al*. [13] considered modeling the τ_l as the arrival times of a homogeneous Poisson process. Unfortunately, this model was not consistent with the clustering of paths observed in their data [10]. Other researchers have considered inhomogeneous Poisson arrivals [2] and doubly-stochastic Poisson arrivals [3]. However, no one has taken the SV/IEEE 802.15.3a model, which includes the clustering effect, and expressed it as a cluster process in a two-dimensional space, as we do in the next two sections.

III. CLUSTER PROCESSES

We give a brief introduction to cluster processes. Cluster processes are superpositions of point processes that can be described by the following two-step procedure. First, cluster "centers" are placed at random locations $X_0, X_1, \ldots$ in a space $\boldsymbol{X}$. Second, conditioned on the value of each X_i, another collection of points $\{Y_{ij}\}_{j=1}^{\infty}$ is placed randomly "near" X_i. In more abstract settings, the points Y_{ij} can belong to a space $\boldsymbol{Y}$ that is different from $\boldsymbol{X}$, but in the SV/IEEE 802.15.3a model, $\boldsymbol{Y} = \boldsymbol{X} = [0,\infty) \times (-\infty,\infty)$, where $[0,\infty)$ is the set in which the path arrival times take values, and $(-\infty,\infty)$ is the set in which the path gains take values.

When $A \subset \boldsymbol{X}$, we let $N_c(A)$ denote the number of cluster centers X_i that lie in A. We can write this mathematically as follows. Let $I_A(x)$ denote the indicator function of A, $I_A(x) := 1$ if $x \in A$ and $I_A(x) := 0$ if $x \notin A$. Then

$$N_c(A) \;:=\; \sum_{i=0}^{\infty} I_A(X_i) \tag{2}$$

counts the number of points X_i that lie in A. We call $N_c(\cdot)$ the **cluster-center process**.

When $B \subset \boldsymbol{Y}$, we let $N_*(B)$ denote the number of Y_{ij} that lie in B, i.e.,

$$N_*(B) \;:=\; \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} I_B(Y_{ij}). \tag{3}$$

We call $N_*(\cdot)$ a **cluster process**.

The number of points from the ith cluster that lie in B is

$$N_r(B|X_i) \;:=\; \sum_{j=1}^{\infty} I_B(Y_{ij}). \tag{4}$$

In the context of the SV/IEEE 802.15.3a model below, for each i, the $\{Y_{ij}\}_{j=1}^{\infty}$ correspond to the (noninitial) rays arriving in the ith multipath cluster. For this reason, we call $N_r(\cdot|X_i)$ the ith **ray process**. We can now write the total number of points in B as the superposition

$$N_*(B) \;=\; \sum_{i=0}^{\infty} N_r(B|X_i).$$

If, as in the SV/IEEE 802.15.3a model, the $N_r(\cdot|x)$ are independent Poisson processes for different x, we see that $N_*(\cdot)$ is conditionally a sum of Poisson processes and therefore conditionally Poisson. In other words, in the SV/IEEE 802.15.3a model, $N_*(\cdot)$ is a **doubly-stochastic Poisson process**.

A. Augmented Cluster Processes

Up to now, the points of the cluster-center process and the points of the ray processes belong to different spaces $\boldsymbol{X}$ and $\boldsymbol{Y}$. In the case of the SV/IEEE 802.15.3a model and its modifications, $\boldsymbol{X} = \boldsymbol{Y}$. It is therefore possible to include the cluster centers as members of their corresponding clusters. Mathematically, we put

$$N(B) \;:=\; N_c(B) + N_*(B).$$

In the SV/IEEE 802.15.3a model, $N_c(\cdot)$ is a Poisson process, and so $N(\cdot)$ is the sum of a Poisson process and a doubly-stochastic Poisson process; note that $N_c(\cdot)$ and $N_*(\cdot)$ are *not* independent since the both depend on the cluster centers X_i. We call $N(\cdot)$ an **augmented cluster process**. Using (2) and (3), we can write

$$N(B) \;=\; \sum_{i=0}^{\infty} I_B(X_i) + \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} I_B(Y_{ij}). \tag{5}$$

If we define $Y_{i0} := X_i$, we can write this more compactly as

$$N(B) \;=\; \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} I_B(Y_{ij}).$$

If we replace $I_B(y)$ by an arbitrary function $\varphi(y)$, we obtain the **counting integral** [11]

$$\int_{\boldsymbol{Y}} \varphi(y)\,N(dy) \;:=\; \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \varphi(Y_{ij}). \tag{6}$$

B. Decomposition of Augmented Cluster Processes

When we define the SV process below, it will be convenient to write (5) as

$$N(B) \;=\; I_B(X_0) + \sum_{j=1}^{\infty} I_B(Y_{0j}) + \sum_{i=1}^{\infty} I_B(X_i) + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} I_B(Y_{ij}).$$

Recalling the definition of N_r in (4), we can rewrite (6) as

$$\begin{aligned}\int_{\boldsymbol{Y}} \varphi(y)\,N(dy) \;&=\; \varphi(X_0) + \int \varphi(y)\,N_r(dy|X_0) \\ &\quad + \sum_{i=1}^{\infty} \varphi(X_i) + \sum_{i=1}^{\infty} \int \varphi(y)\,N_r(dy|X_i).\end{aligned} \tag{7}$$

Now, the easiest way to construct a cluster process is to take the cluster sequences $\{Y_{ij}\}_{j=1}^{\infty}$ to be independent (as a function of i). This means that for distinct values of x, the processes $N_r(\cdot|x)$ are independent. In the SV/IEEE 802.15.3a model, not only do we do have this, but we also have the initial cluster center X_0 independent of the remaining cluster centers $\{X_i\}_{i=1}^{\infty}$. Furthermore, X_0 is such that $N_r(\cdot|X_0)$ does not depend on X_0. Hence, for the SV/IEEE 802.15.3a model, counting integrals with respect to $N(\cdot)$ can be written as the sum of the three statistically independent quantities,

$$\varphi(X_0), \quad \Phi_{r0} \;:=\; \int \varphi(y)\,N_r(dy|X_0),$$

and

$$\Phi_{\otimes} \;:=\; \sum_{i=1}^{\infty} \varphi(X_i) + \sum_{i=1}^{\infty} \int \varphi(y)\,N_r(dy|X_i).$$

Because these three parts are independent, they can be analyzed separately.

IV. The SV/IEEE 802.15.3a Augmented Cluster Process

In the SV multipath channel model [10], paths arrive in clusters.

A. Distribution of the Initial Paths of the Clusters

The initial path in the initial cluster arrives at time zero with a gain G_{00} that has a density that we denote by $f_{0,0}(\cdot)$.

Next, independent of G_{00}, the arrival times of the initial paths of the remaining clusters are modeled as a homogeneous Poisson process whose rate we denote by C. If the initial path of such a cluster arrives at time τ, its gain has a density that we denote by $f_{\tau,\tau}(\cdot)$. The gains of different initial paths are independent and can be considered marks of the Poisson arrival times. Such a marked Poisson process is equivalent to a two-dimensional Poisson process with intensity function [6, Sec. 5.2]

$$\lambda_1(\tau,\gamma) \;:=\; C f_{\tau,\tau}(\gamma), \quad \tau \geq 0,\ \gamma \in \mathbb{R}. \tag{8}$$

For future reference, on functions $\varphi(\tau,\gamma)$, we define the linear functional

$$\bar{\Lambda}_1\varphi \;:=\; \int_0^{\infty}\int_{-\infty}^{\infty} \varphi(\tau,\gamma)\lambda_1(\tau,\gamma)\,d\gamma\,d\tau. \tag{9}$$

B. Distribution of Noninitial Paths of the Clusters

Conditional on the arrival times of the initial paths of the clusters, the arrival times of the noninitial paths in different clusters, including the cluster that starts at time zero, are modeled as independent homogeneous Poisson processes. Each of these Poisson processes has the same rate, which we denote by R. If one of these paths arrives at time s and is part of a cluster that started at time τ, the arriving path gain has a density that we denote by $f_{\tau,s}(\cdot)$. These gains are independent and can be considered marks of the Poisson arrival times of the noninitial paths in the cluster. As mentioned above, such a marked Poisson process is equivalent to a two-dimensional Poisson process. Here the intensity function is[1]

$$\lambda_r(s,g|\tau,\gamma) \;:=\; R f_{\tau,s}(g) I_{[\tau,\infty)}(s). \tag{10}$$

Note that $\lambda_r(s,g|\tau,\gamma)$ depends on τ but not γ.

For future reference, on functions $\varphi(s,g)$, we define the operator

$$\begin{aligned}(\bar{\Lambda}_r\varphi)(\tau,\gamma) \;&:=\; \int_0^{\infty}\int_{-\infty}^{\infty} \varphi(s,g)\lambda_r(s,g|\tau,\gamma)\,dg\,ds \\ &=\; R\int_{\tau}^{\infty}\int_{-\infty}^{\infty} \varphi(s,g)\,f_{\tau,s}(g)\,dg\,ds,\end{aligned} \tag{11}$$

which does not depend on γ. For this reason, we sometimes write $(\bar{\Lambda}_r\varphi)(\tau)$ to denote (11).

C. Assumptions about the Densities of the Gains

Following Saleh and Valenzuela [10, eq. (26)] and Batra *et al.* [1, p. 2126], we assume $f_{\tau,s}(\cdot)$ has a second moment proportional to

$$e^{-\tau/\tau_0}e^{-(s-\tau)/s_0}, \tag{12}$$

[1]Since the intensity in (10) is zero for $s < \tau$, the Poisson process starts at time τ. This is in contrast to [1] and [10]. Their ray processes were defined by taking Poisson processes starting at time zero and then translating them by the arrival time of the initial path in the cluster. The two constructions are equivalent provided we adjust the definition of $f_{\tau,s}(\cdot)$. This is done in (12) where we use $s-\tau$; [1] and [10] would use only s.

where τ_0 and s_0 are power-delay time constants. In [10], $f_{\tau,s}(\cdot)$ is taken to be a Rayleigh density. For the IEEE 802.15.3a model in [1], a $\{\pm 1\}$-valued-Bernoulli$(1/2)$ mixture of log-normal densities is used. This implies that under the model in [1], $f_{\tau,s}(\cdot)$ is even and therefore has zero mean. Note that our second-order results in the next Section make no assumptions about the density $f_{\tau,s}(\cdot)$ other than its having zero mean and second moment proportional to (12).

V. Some Statistics of the IEEE 802.15.3a Model

Let $\{(S_{i0}, G_{i0})\}_{i=1}^{\infty}$ be the points of the two-dimensional Poisson process with intensity function λ_1 in (8). Put $X_i := (S_{i0}, G_{i0})$ for $i \geq 1$, and put $X_0 := (0, G_{00})$, where G_{00} has density $f_{00}(\cdot)$ and is independent of the Poisson process. For $i = 0, 1, \ldots$, given $X_i = (\tau, \gamma)$, let $\{(S_{ij}, G_{ij})\}_{j=1}^{\infty}$ be the points of the two-dimensional Poisson process with intensity function $\lambda_r(\cdot,\cdot|\tau,\gamma)$ in (10). Put $Y_{ij} := (S_{ij}, G_{ij})$ for $j \geq 1$, and put $Y_{i0} := X_i$. Then the counting integral in (6) becomes

$$\Phi := \int_0^\infty \int_{-\infty}^\infty \varphi(s,g)\, N(ds \times dg) = \sum_{i=0}^\infty \sum_{j=0}^\infty \varphi(S_{ij}, G_{ij}). \quad (13)$$

Here are some choices of $\varphi(s,g)$ that make counting integrals so valuable:

- *Number of Paths in a Time Window*. If we take $\varphi(s,g) = I_{[a,b]}(s)$, then (13) becomes
$$\sum_{i=0}^\infty \sum_{j=0}^\infty I_{[a,b]}(S_{ij}).$$
This is the number of arrival times S_{ij} that fall in the time window $[a,b]$.
- *Sum of Path Gains Arriving in a Time Window*. If $\varphi(s,g) = g I_{[a,b]}(s)$, then (13) becomes
$$\sum_{i=0}^\infty \sum_{j=0}^\infty G_{ij} I_{[a,b]}(S_{ij}).$$
This adds up all the path gains G_{ij} whose path arrival time S_{ij} falls in the time window $[a,b]$.
- *Received UWB Waveforms*. If we take $\varphi(s,g) = g\xi(t-s)$, then (13) becomes
$$\sum_{i=0}^\infty \sum_{j=0}^\infty G_{ij}\xi(t - S_{ij}).$$
This is exactly the received UWB waveform $\rho(t)$ in (1) modulo a renumbering of delays and gains to accommodate double subscripts. Here of course, the joint distributions of the delays and gains are completely specified.

The key to analyzing Φ in (13) is to break it into the sum of the three independent terms,

$$\Phi \;=\; \varphi(0, G_{00}) + \Phi_{r0} + \Phi_{\otimes},$$

as discussed in Section III-B. If $\psi(s,g)$ is another function, then we can

$$\Psi \;=\; \psi(0, G_{00}) + \Psi_{r0} + \Psi_{\otimes}$$

where these terms are independent and defined analogously. Furthermore, the *non*corresponding terms of Φ and Ψ are also independent.

A. Mean and Covariance of Number of Paths

Let $0 \leq a < b \leq c < d$, and put $\varphi(s,g) = I_{[a,b]}(s)$ and $\psi(s,g) = I_{[c,d]}(s)$. Then Φ is the number of multipath arrivals in the time window $[a,b]$, and Ψ is the number of multipath arrivals in the time window $[c,d]$. Using (27), (29), and (33) in Appendix C and the simplifications discussed there, it can be shown that

$$\mathsf{E}[\Phi] \;=\; I_{[a,b]}(0) + R(b-a) + C(b-a)\left[1 + R\frac{b+a}{2}\right]. \quad (14)$$

Similarly, using (30) and (35), it can be shown that

$$\begin{aligned}\mathsf{var}(\Phi) \;=\; & R(b-a) \\ & + C(b-a)\left[1 + R\frac{3b-a}{2} + R^2(b-a)\left(\frac{b+2a}{3}\right)\right].\end{aligned}$$

As for the covariance of the number of paths in disjoint intervals, first write

$$\begin{aligned}\mathsf{cov}(\Phi,\Psi) \;=\; & \mathsf{cov}(\varphi(0,G_{00}), \psi(0,G_{00})) + \mathsf{cov}(\Phi_{r0}, \Psi_{r0}) \\ & + \mathsf{cov}(\Phi_{\otimes}, \Psi_{\otimes}).\end{aligned}$$

The first term on the right is zero since the intervals $[a,b]$ and $[c,d]$ are disjoint. The second term is zero by the paragraph containing (30). As for $\mathsf{cov}(\Phi_{\otimes}, \Psi_{\otimes})$, which is given by (34), it is argued at the end of Appendix C that the first, second, and fourth terms of (34) are zero. Evaluating the remaining terms, it can be shown that

$$\mathsf{cov}(\Phi,\Psi) \;=\; CR(b-a)(d-c)[1 + R(b+a)/2].$$

B. Mean and Covariance of Sum of Path Gains in a Time Window

Let $0 \leq a < b \leq c < d$ as above, and put $\varphi(s,g) = g I_{[a,b]}(s)$ and $\psi(s,g) = g I_{[c,d]}(s)$. Then Φ is the sum of the gains of the multipath arrivals in the time window $[a,b]$, and Ψ is the sum of the gains of the multipath arrivals in the time window $[c,d]$. Using (27), (29), and (33) along with the fact that $f_{\tau,s}(\cdot)$ has zero mean, it is easy to show that

$$\mathsf{E}[\Phi] \;=\; 0.$$

Using (30), and again using the fact that $(\bar{\Lambda}_r \varphi)(\tau) = 0$ in (35), we find that

$$\begin{aligned}\mathsf{var}(\Phi) \;=\; & I_{[a,b]}(0) E[G_{00}^2] + R\int_a^b \mathsf{E}_{0,s}[G^2]\, ds \\ & + \bar{\Lambda}_1(\varphi^2) + \bar{\Lambda}_1(\bar{\Lambda}_r(\varphi^2)), \end{aligned} \quad (15)$$

where G is a generic random variable with density $f_{0,s}(\cdot)$. On account of (12), (15) is computable in closed form. For example, the integral term is[2]

$$R s_0 (e^{-a/s_0} - e^{-b/s_0}), \quad (16)$$

and

$$\bar{\Lambda}_1(\varphi^2) \;=\; C\tau_0 (e^{-a/\tau_0} - e^{-b/\tau_0}). \quad (17)$$

[2]Recall that $\mathsf{E}_{\tau,s}[G^2]$ is *proportional* to (12). In (16)–(20), we have taken this constant of proportionality to be one.

A bit more work shows that the last term in (15) can also be expressed in closed form. It is

$$\begin{aligned}\overline{\Lambda}_1(\overline{\Lambda}_r(\varphi^2)) &= CR\big[\zeta(a,b,s_0)\zeta\big(0,a,s_0\tau_0/(s_0-\tau_0)\big) \\ &\quad + s_0\zeta(a,b,\tau_0) \\ &\quad - s_0\zeta\big(a,b,s_0\tau_0/(s_0-\tau_0)\big)e^{-b/s_0}\big], \quad (18)\end{aligned}$$

where

$$\zeta(a,b,\mu) := \mu[e^{-a/\mu}-e^{-b/\mu}].$$

If we put $a=0$ and let $b\to\infty$, we find that

$$\lim_{b\to\infty}\mathsf{var}(\Phi) = 1+Rs_0+C\tau_0+CRs_0\tau_0. \quad (19)$$

Example 1: Suppose that we take $a=0$ in the definition of $\varphi(s,g)=gI_{[a,b]}(s)$. We can then plot (15) as a function of b as shown in Fig. 2. For the plot we have used the parameters of channel model CM3 in [1, Table II], namely

$$C=0.0667, \quad R=2.1, \quad \tau_0=14.0, \quad \text{and} \quad s_0=7.9.$$

Hence, the limiting value of $\mathsf{var}(\Phi)$ is about 34. At about $b=33$ ns, the variance has reached 90% of the limiting value. Ninety percent of the limiting value is about 31. Note that by (14), the expected number of paths in the time window $[0,33]$ is about 149. We also point out that the expected number of multipath clusters in $[a,b]$ is given by

$$\begin{aligned}\mathsf{E}[N_c([a,b]\times\mathrm{I\!R})] &= \mathsf{E}[I_{[a,b]}(0)+N_1([a,b]\times\mathrm{I\!R})] \\ &= I_{[a,b]}(0)+\Lambda_1([a,b]\times\mathrm{I\!R}) \\ &= I_{[a,b]}(0)+C(b-a).\end{aligned}$$

The expected number of clusters in $[0,33]$ is about 3.

We conclude this subsection by writing

$$\begin{aligned}\mathsf{cov}(\Phi,\Psi) &= \mathsf{cov}(\varphi(0,G_{00}),\psi(0,G_{00}))+\mathsf{cov}(\Phi_{r0},\Psi_{r0}) \\ &\quad +\mathsf{cov}(\Phi_\otimes,\Psi_\otimes).\end{aligned}$$

The first term on the right is zero since the intervals $[a,b]$ and $[c,d]$ are disjoint. The second term is zero by the paragraph containing (30). As for $\mathsf{cov}(\Phi_\otimes,\Psi_\otimes)$, it is argued at the end of Appendix C that the first, second, and fourth terms of (34) are zero. However, the remaining terms involve $\overline{\Lambda}_r\psi$, which is zero since $\mathsf{E}_{\tau,s}[G]=0$. Thus,

$$\mathsf{cov}(\Phi,\Psi) = 0,$$

and we see that the sums of gains in different time windows are uncorrelated.

Remark: If we are interested in the sum of the *squares of* the gains in an interval, we can take $\widetilde{\varphi}(s,g)=g^2I_{[a,b]}(s)$. If we then use the formulas of Appendix C to compute $\mathsf{E}[\widetilde{\Phi}]$, we get exactly (15). This is easy to see if we note that since the square of an indicator is equal to itself, $\widetilde{\varphi}(s,g)=g^2I_{[a,b]}(s)=\varphi(s,g)^2$; e.g., $\overline{\Lambda}_1(\varphi^2)=\overline{\Lambda}_1\widetilde{\varphi}$. Now, since the limiting value of (15) is given by (19), we see that the expected sum of squares of the gains in $[0,b]$ converges to (19). Thus, although the expected number of paths in $[0,b]$ grows quadratically in b by (14), the energies of the paths decay so rapidly that the expected sum of squares of gains over $[0,b]$ levels off for large b.

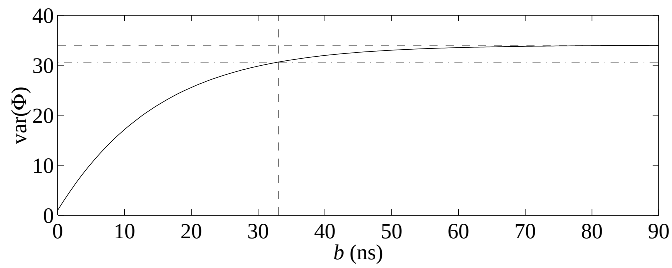

Fig. 2. Plot of $\mathsf{var}(\Phi)$ in (15) as a function of b with $a=0$. The limiting value of the curve is about 34. Ninety percent of this value is about 31 and is reached at about $b=33$ ns.

C. Mean and Covariance of the UWB Channel Response

If we take $\varphi(s,g)=g\xi(t_1-s)$ and $\psi(s,g)=g\xi(t_2-s)$, then $\Phi=\rho(t_1)$ and $\Psi=\rho(t_2)$, where $\rho(t)$ is the UWB waveform seen at the receiver. Using the formulas in Appendix C, it is not hard to show that

$$\mathsf{E}[\rho(t_1)] = 0,$$

and

$$\begin{aligned}&\mathsf{cov}(\rho(t_1),\rho(t_2)) \\ &\quad = \mathsf{E}[G_{00}^2]\xi(t_1)\xi(t_2) \\ &\qquad +R\int_0^\infty \xi(t_1-s)\xi(t_2-s)\mathsf{E}_{0,s}[G^2]\,ds \\ &\qquad +C\int_0^\infty \xi(t_1-\tau)\xi(t_2-\tau)\mathsf{E}_{\tau,\tau}[G^2]\,d\tau \\ &\qquad +CR\int_0^\infty\int_\tau^\infty \xi(t_1-s)\xi(t_2-s)\mathsf{E}_{\tau,s}[G^2]\,ds\,d\tau.\end{aligned}$$

Note that this last double integral can be reduced to a single integral by changing the order of integration and using (12).

Example 2: Let us compute the expected energy in the received waveform $\rho(t)$. An easy calculation shows that

$$\begin{aligned}\mathsf{E}\Big[\int_{-\infty}^\infty \rho(t)^2\,dt\Big] &= \int_{-\infty}^\infty \mathsf{E}[\rho(t)^2]\,dt \\ &= \int_{-\infty}^\infty \mathsf{cov}(\rho(t),\rho(t))\,dt \\ &= \|\xi\|^2\big\{1+Rs_0+C\tau_0+CRs_0\tau_0\big\}, \quad (20)\end{aligned}$$

where $\|\xi\|^2 := \int_{-\infty}^\infty \xi(t)^2\,dt$ is the energy in the transmitted waveform. Notice that (20) is proportional to (19). For the situation in Example 1 and Fig. 2, we see that 90% of the received signal energy is due to the gains of the paths that arrive in the first 33 ns.

VI. Conclusion

We have presented a careful development of the SV/IEEE 802.15.3a model as an augmented cluster point process that is composed of three statistically independent components that can be separately analyzed. We have shown that important quantities of interest can be expressed as shot-noise random variables driven by the augmented cluster process, and we have derived formulas for means and variances of such shot-noise random variables. Although the general formulas involve double or even triple integrals, in many cases, they can be expressed in closed form or simplified to involve only single or double integral.

APPENDIX A
POISSON-DRIVEN SHOT-NOISE RANDOM VARIABLES

Let $N_1(\cdot)$ be a Poisson process on a space $\boldsymbol{X}$ [6, Ch. 2]. In other words, for $A \subset \boldsymbol{X}$, $N_1(A)$ denotes the number of points in A. The random variable $N_1(A)$ is Poisson, and we denote its mean value by

$$\Lambda_1(A) \;:=\; \mathsf{E}[N_1(A)].$$

When $N_1(A)$ is regarded as a function of A, $N_1(\cdot)$ is a nonnegative, integer-valued measure. When $\Lambda_1(\cdot)$ is regarded as a function of A, we call $\Lambda_1(\cdot)$ the **mean measure** of $N_1(\cdot)$.

If v is a function on $\boldsymbol{X}$, we define the shot-noise random variable [6, Ch. 3]

$$V \;:=\; \int v(x)\,N_1(dx). \tag{21}$$

Then

$$\mathsf{E}[V] \;=\; \int v(x)\,\Lambda_1(dx),$$

where we assume $v \in L^1(\Lambda_1)$ both here and in (21) in order that the integrals be well defined. If we also have $w \in L^1(\Lambda_1)$ and define

$$W \;:=\; \int w(x)\,N_1(dx),$$

then

$$\begin{aligned}\mathsf{E}[VW] \;&=\; \int v(x)w(x)\,\Lambda_1(dx)\\ &\quad+\left[\int v(x)\,\Lambda_1(dx)\right]\left[\int w(x)\,\Lambda_1(dx)\right],\end{aligned}$$

where we additionally assume $v,w \in L^2(\Lambda_1)$. We also have the moment generating function

$$\mathsf{E}[e^{sV}] \;=\; \exp\left[\int [e^{sv(x)}-1]\,\Lambda_1(dx)\right],$$

where s is complex and $[e^{sv(\cdot)}-1] \in L^1(\Lambda_1)$.

In order to write the preceding expectations in a more compact form, we define the linear functional

$$\overline{\Lambda}_1 v \;:=\; \int v(x)\Lambda_1(dx) \tag{22}$$

so that

$$\mathsf{E}[V] \;=\; \overline{\Lambda}_1 v, \quad \mathsf{E}[VW] \;=\; \overline{\Lambda}_1(v\cdot w) + \overline{\Lambda}_1 v\cdot\overline{\Lambda}_1 w,$$

and

$$\mathsf{E}[e^{sV}] \;=\; \exp\big[\overline{\Lambda}_1\big(e^{sv(\cdot)}-1\big)\big].$$

APPENDIX B
ANALYSIS OF INTEGRALS WITH RESPECT TO $N_1(\cdot)$ AND $N_\times(\cdot)$

Doubly-Poisson Cluster Processes

Let $N_1(\cdot)$ be the Poisson process defined in Appendix A with mean measure $\Lambda_1(\cdot)$ and the linear functional $\overline{\Lambda}_1$ defined in (22). Let $\{N_r(\cdot|x), x \in \boldsymbol{X}\}$ be a family of independent Poisson processes on a set $\boldsymbol{Y}$. Assume the $N_r(\cdot|x)$ are independent of $N_1(\cdot)$. Denote the mean measure of $N_r(\cdot|x)$ by $\Lambda_r(\cdot|x)$.

We now define the cluster process $N_\times(\cdot)$ on $\boldsymbol{Y}$ by [4, Ch. 8]

$$N_\times(B) \;:=\; \int N_r(B|x)\,N_1(dx), \quad B \subset \boldsymbol{Y}.$$

Because $N_1(\cdot)$ is a Poisson process, $N_\times(\cdot)$ is called a **Poisson cluster process** [4]. Since $N_r(\cdot|x)$ is also Poisson, we call $N_\times(\cdot)$ a **doubly-Poisson cluster process**. If we let $\mathcal{N}_1$ denote the σ-field generated by $N_1(\cdot)$, then conditioned on $\mathcal{N}_1$, $N_\times(\cdot)$ is a sum of independent Poisson processes. Hence, conditioned on $\mathcal{N}_1$, the mean measure of $N_\times(\cdot)$ is

$$M_\times(B) \;:=\; \mathsf{E}[N_\times(B)|\mathcal{N}_1] \;=\; \int \Lambda_r(B|x)\,N_1(dx).$$

In other words, because the $N_r(\cdot|x)$ are independent Poisson processes independent of $N_1(\cdot)$, $N_\times(\cdot)$ a doubly-stochastic Poisson process with conditional mean measure $M_\times(\cdot)$. Thus, $N_\times(\cdot)$ is both a cluster process and a doubly-stochastic Poisson process.

Doubly-Poisson-Cluster-Driven Shot-Noise Random Variables

Let p and q be functions defined on $\boldsymbol{Y}$, and introduce the cluster-process-driven shot-noise random variables

$$P \;:=\; \int p(y)\,N_\times(dy) \quad\text{and}\quad Q \;:=\; \int q(y)\,N_\times(dy).$$

Then

$$\mathsf{E}[P|\mathcal{N}_1] = \int p(y)\,M_\times(dy) = \int\left[\int p(y)\,\Lambda_r(dy|x)\right]N_1(dx).$$

It is now convenient to introduce the operator notation

$$(\overline{\Lambda}_r p)(x) \;:=\; \int p(y)\,\Lambda_r(dy|x)$$

so that we can write

$$\mathsf{E}[P|\mathcal{N}_1] \;=\; \int p(y)\,M_\times(dy) \;=\; \int(\overline{\Lambda}_r p)(x)\,N_1(dx), \tag{23}$$

which we recognize as a Poisson-driven shot-noise random variable analogous to (21). A similar argument shows that

$$\begin{aligned}\mathsf{E}[PQ|\mathcal{N}_1] \;&=\; \int p(y)q(y)\,M_\times(dy)\\ &\quad+\left[\int p(y)\,M_\times(dy)\right]\left[\int q(y)\,M_\times(dy)\right]\\ &=\; \int(\overline{\Lambda}_r p\cdot q)(x)\,N_1(dx)\\ &\quad+\left[\int(\overline{\Lambda}_r p)(x)\,N_1(dx)\right]\left[\int(\overline{\Lambda}_r q)(x)\,N_1(dx)\right].\end{aligned} \tag{24}$$

Since the integrals in (23) and (24) are Poisson-driven shot-noise random variables, taking expectations yields

$$\mathsf{E}[P] \;=\; \overline{\Lambda}_1(\overline{\Lambda}_r p)$$

and

$$\begin{aligned}\mathsf{E}[PQ] \;&=\; \overline{\Lambda}_1(\overline{\Lambda}_r(p\cdot q)) + \overline{\Lambda}_1((\overline{\Lambda}_r p)\cdot(\overline{\Lambda}_r q))\\ &\quad+\overline{\Lambda}_1(\overline{\Lambda}_r p)\cdot\overline{\Lambda}_1(\overline{\Lambda}_r q).\end{aligned}$$

We also need that

$$\begin{aligned}\mathsf{E}[VP|\mathcal{N}_1] \;&=\; V\,\mathsf{E}[P|\mathcal{N}_1]\\ &=\; V\int(\overline{\Lambda}_r p)(x)\,N_1(dx),\end{aligned}$$

which is a product of Poisson-driven shot-noise random variables. Hence,

$$\mathsf{E}[VP] \;=\; \overline{\Lambda}_1(v\cdot(\overline{\Lambda}_r p)) + \overline{\Lambda}_1(v)\cdot\overline{\Lambda}_1(\overline{\Lambda}_r p).$$

Example 3: Using the foregoing formulas, it is easy to see that

$$\mathsf{E}[V+P] \;=\; \overline{\Lambda}_1(v)+\overline{\Lambda}_1(\overline{\Lambda}_r p), \tag{25}$$

and

$$\begin{aligned}\mathsf{cov}(V+P,W+Q) = {} & \overline{\Lambda}_1(v\cdot w)+\overline{\Lambda}_1(w\cdot\overline{\Lambda}_r p)+\overline{\Lambda}_1(v\cdot\overline{\Lambda}_r q)\\ & +\overline{\Lambda}_1(\overline{\Lambda}_r(p\cdot q))+\overline{\Lambda}_1((\overline{\Lambda}_r p)\cdot(\overline{\Lambda}_r q)).\end{aligned} \tag{26}$$

APPENDIX C
ANALYSIS OF SV/IEEE 802.15.3A COUNTING INTEGRALS

As discussed in Section III-B, in the SV/IEEE 802.15.3a model, counting integrals Φ such as (7) and (13) can be written as the sum of three statistically independent terms, $\Phi = \varphi(X_0)+\Phi_{r0}+\Phi_{\otimes}$. We now derive general formulas for some statistics of $\varphi(X_0)$, Φ_{r0}, and $\Phi_{\otimes}$. Motivated by the specific counting integrals in Section V, we restrict attention to integrands $\varphi(s,g)$ of product form, say $\varphi_1(s)\varphi_2(g)$.

The First Component

Under the SV/IEEE 802.15.3a model, $\varphi(X_0)=\varphi(0,G_{00})$. For a product-form integrand, this becomes $\varphi_1(0)\varphi_2(G_{00})$. Hence,

$$\mathsf{E}[\varphi(X_0)] \;=\; \varphi_1(0)\mathsf{E}[\varphi_2(G_{00})]. \tag{27}$$

If $\varphi(s)=I_{[a,b]}(s)$ and $a>0$, then this expectation is zero. If $\varphi_2(g)$ is equal to g or g^2 and $f_{0,0}$ is Rayleigh or the IEEE 802.15.3a lognormal mixture, this expectation is available in closed form.

The Second Component

We next consider

$$\begin{aligned}\Phi_{r0} \;&:=\; \int \varphi(y)\,N_r(dy|X_0)\\ &=\; \int_0^\infty\int_{-\infty}^\infty \varphi(s,g)\,N_r(ds\times dg|0,G_{00}).\end{aligned} \tag{28}$$

Since $N_r(\cdot|0,G_{00})$ does not depend on G_{00}, the above right-hand side is actually a Poisson-driven shot-noise random variable. Such random variables are discussed in Appendix A, where formulas for their mean, variance, and moment generating function are given. In the case at hand,

$$\mathsf{E}[\Phi_{r0}] \;=\; (\overline{\Lambda}_r\varphi)(0),$$

where $\overline{\Lambda}_r$ was defined in (11). If φ has product form, we can write

$$\mathsf{E}[\Phi_{r0}] \;=\; (\overline{\Lambda}_r\varphi)(0) \;=\; R\int_0^\infty \varphi_1(s)\mathsf{E}_{0,s}[\varphi_2(G)]\,ds, \tag{29}$$

where G is a generic random variable with density $f_{0,s}(\cdot)$. For φ_2 a polynomial, the expectation will typically be available in closed form, usually as an exponential in s (cf. (12)); if we also have $\varphi_1(s)=I_{[a,b]}(s)$, then $(\overline{\Lambda}_r\varphi)(0)$ can be computed in closed form.

Analogous to (28), suppose we have another integral Ψ_{r0} of a function $\psi(s,g)$ also of product form. Then the covariance between these two Poisson-driven counting integrals is

$$\begin{aligned}\mathsf{cov}(\Phi_{r0},\Psi_{r0}) \;&=\; (\overline{\Lambda}_r(\varphi\cdot\psi))(0)\\ &=\; R\int_0^\infty \varphi_1(s)\psi_1(s)\mathsf{E}_{0,s}[\varphi_2(G)\psi_2(G)]\,ds.\end{aligned}$$

If φ_1 and ψ_1 are indicator functions of disjoint intervals, then the covariance is zero. If the intervals are the same, say $[a,b]$, then

$$\mathsf{cov}(\Phi_{r0},\Psi_{r0}) \;=\; R\int_a^b \mathsf{E}_{0,s}[\varphi_2(G)\psi_2(G)]\,ds. \tag{30}$$

As noted above, there are interesting cases in which this expectation and integral will be computable in closed form.

The Third Component

We now focus on the properties of

$$\sum_{i=1}^\infty \varphi(X_i)+\sum_{i=1}^\infty\int \varphi(y)\,N_r(dy|X_i). \tag{31}$$

If we put

$$N_1(B) \;:=\; \sum_{i=1}^\infty I_B(X_i) \quad\text{and}\quad N_\times(B) \;:=\; \sum_{i=1}^\infty N_r(B|X_i),$$

then (31) can be written as

$$\int \varphi(x)\,N_1(dx)+\int \varphi(y)N_\times(dy).$$

Properties of this sum of counting integrals are derived in Appendix B. Hence, if we put

$$N_\otimes(\cdot) \;:=\; N_1(\cdot)+N_\times(\cdot),$$

then the counting integral

$$\Phi_\otimes \;:=\; \int_0^\infty\int_{-\infty}^\infty \varphi(s,g)\,N_\otimes(ds\times dg). \tag{32}$$

can be written as $\Phi_\otimes = V+P$, where V and P are the corresponding integrals with respect to $N_1(\cdot)$ and $N_\times(\cdot)$. Since $\mathsf{E}[V+P]$ is given by eq. (25) in Appendix B, we have

$$\mathsf{E}[\Phi_\otimes] \;=\; \overline{\Lambda}_1(\varphi)+\overline{\Lambda}_1(\overline{\Lambda}_r\varphi), \tag{33}$$

where $\overline{\Lambda}_1$ is the linear functional defined in (9) and $\overline{\Lambda}_r$ is the linear operator defined in (11).

Now suppose that

$$\Psi_\otimes \;:=\; \int_0^\infty\int_{-\infty}^\infty \psi(s,g)\,N_\otimes(ds\times dg).$$

If we similarly write $\Psi_\otimes = W+Q$, then

$$\mathsf{cov}(\Phi_\otimes,\Psi_\otimes) \;=\; \mathsf{cov}(V+P,W+Q)$$

is given by eq. (26) in Appendix B. Thus,

$$\begin{aligned}\mathsf{cov}(\Phi_\otimes,\Psi_\otimes) \;=\; & \overline{\Lambda}_1(\varphi\cdot\psi)+\overline{\Lambda}_1(\psi\cdot\overline{\Lambda}_r\varphi)+\overline{\Lambda}_1(\varphi\cdot\overline{\Lambda}_r\psi)\\ & +\overline{\Lambda}_1(\overline{\Lambda}_r(\varphi\cdot\psi))+\overline{\Lambda}_1((\overline{\Lambda}_r\varphi)\cdot(\overline{\Lambda}_r\psi)).\end{aligned} \tag{34}$$

We also record the special case,

$$\begin{aligned}\mathsf{var}(\Phi_\otimes) \;=\; & \overline{\Lambda}_1(\varphi^2)+2\overline{\Lambda}_1(\varphi\cdot\overline{\Lambda}_r\varphi)\\ & +\overline{\Lambda}_1(\overline{\Lambda}_r(\varphi^2))+\overline{\Lambda}_1((\overline{\Lambda}_r\varphi)^2).\end{aligned} \tag{35}$$

Simplifications of (33)–(35): The assumptions of the IEEE 802.15.3a model allow for several further simplifications in the formulas for $\bar{\Lambda}_1$ and $\bar{\Lambda}_r$. This makes the computation of (33)–(35) quite tractable.

We first consider $\bar{\Lambda}_r$. Since $\lambda_r(s,g|\tau,\gamma)$ is defined in (10) not to depend on γ, we write

$$(\bar{\Lambda}_r\varphi)(\tau) \;=\; \int_0^\infty \int_{-\infty}^\infty \varphi(s,g)\lambda_r(s,g|\tau,\gamma)\,ds\,dg.$$

Next, since we will be concerned only with functions φ of product form, e.g., $\varphi(s,g) = \varphi_1(s)\varphi_2(g)$, we can further exploit the definition of λ_r to write

$$(\bar{\Lambda}_r\varphi)(\tau) \;=\; R\int_\tau^\infty \varphi_1(s)\mathsf{E}_{\tau,s}[\varphi_2(G)]\,ds,$$

where G is a generic random variable with density $f_{\tau,s}$ mentioned in (10). Note that if $\varphi_1(s) = I_{[a,b]}(s)$ for some $0 \le a < b$, then $(\bar{\Lambda}_r\varphi)(\tau) = 0$ for $\tau > b$. Also, since G has an even density, if φ_2 is odd, $(\bar{\Lambda}_r\varphi)(\tau) = 0$.

We next consider $\bar{\Lambda}_1$. If ψ also has product form, and if we exploit the definition of λ_1 in (8), then

$$\bar{\Lambda}_1(\varphi\cdot\psi) \;=\; C\int_0^\infty \varphi_1(\tau)\psi_1(\tau)\mathsf{E}_{\tau,\tau}[\varphi_2(\Gamma)\psi_2(\Gamma)]\,d\tau,$$

where Γ is a generic random variable with density $f_{\tau,\tau}$ mentioned in (8). Note that if $0 \le a < b \le c < d$, and $\varphi_1(s) = I_{[a,b]}(s)$ and $\psi_1(s) = I_{[c,d]}(s)$, then $\varphi\cdot\psi = 0$ and the first and fourth terms in (34) are zero.

We can now write

$$\bar{\Lambda}_1(\psi\cdot\bar{\Lambda}_r\varphi) \;=\; C\int_0^\infty \psi_1(\tau)(\bar{\Lambda}_r\varphi)(\tau)\mathsf{E}_{\tau,\tau}[\psi_2(\Gamma)]\,d\tau.$$

Note that if φ_1 and ψ_1 are indicators of disjoint intervals as above, then the second term in (34) is zero. Also, since Γ has an even density, if ψ_2 is odd, $\bar{\Lambda}_1(\psi\cdot\bar{\Lambda}_r\varphi) = 0$.

For the calculations we consider here, the expectations $E_{\tau,\tau}$ and $E_{\tau,s}$ will always be available in closed form. Hence, the first term in (33) and in (34) involve at most one integral. The second term in (33) and the second through fourth terms in (34) require at most a double integral. The last term in (34) may require a triple integral, unless $\varphi_2 = \psi_2$, in which case at most a double integral is necessary. In some cases, all terms can be computed in closed form.

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